MULTIPLE MODEL ADAPTIVE ESTIMATION
FOR
TIME SERIES ANALYSIS

THESIS

Ibrahim DULGER, First Lieutenant, TUAF

AFIT/GOR/ENS/01M-07

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GOR/ENS/01M-07

MULTIPLE MODEL ADAPTIVE ESTIMATION

FOR

TIME SERIES ANALYSIS

THESIS

Presented to the Faculty

Department of Operational Sciences

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Operations Research

Ibrahim DULGER

First Lieutenant, TUAF, B.S.

March 2001

AFIT/GOR/ENS/01M-07

MULTIPLE MODEL ADAPTIVE ESTIMATION

FOR

TIME SERIES ANALYSIS

Ibrahim DULGER
First Lieutenant, TUAF

Approved:

Mark A. Gallagher   23 Feb 01

Mark A. Gallagher, Lt Col, USAF (Advisor)   date

Jeffrey W. Lanning   23 Feb 01

Jeffrey W. Lanning, Major, USAF (Reader)   date

ACKNOWLEDGEMENTS

I sincerely value my thesis advisor, Lt. Col. Mark Gallagher, for his guidance, and support throughout the course of this thesis effort. Without his insight, experience, and knowledge I could not accomplish this thesis. He has always been there; helpful, patient and understanding whenever I felt desperate, frustrated and full of questions. He always had answers, which made my life easier. I would like to thank my reader, Maj. Jeffrey Lanning, for providing his valuable comments.

I would like to thank to my fellow Turkish friends either at the AFIT or in Dayton area, for not making me homesick in a foreign country. I also thank to all instructors and students of the AFIT, International Military Student's Office and other international friends for their hospitality, and understanding.

I am very grateful to Turkish Air Force and great Turkish Nation for providing me such a wonderful education and to my family for their beliefs in my success.

I express my deepest gratitude to my wife. She persevered through this long and endless effort. Her unfailing support and encouragement made all of this possible.

Ibrahim DULGER

TABLE OF CONTENTS

Page

LIST OF FIGURES

LIST OF TABLES

AFIT/GOR/ENS/01M-07

ABSTRACT

Multiple Model Adaptive Estimation (MMAE) is a Bayesian technique that applies a bank of Kalman filters to predict future observations. Each Kalman filter is based on a different set of parameters and hence produces different residuals. The likelihood of each Kalman filter's prediction is determined by a magnitude of the residuals. Since some researchers have obtained good forecasts using a single Kalman filter, we tested MMAE's ability to make time series predictions. Our Kalman filters have a dynamics model based on a Box-Jenkins Auto-Regressive Moving Average (ARMA) model and a measure model with additive noise. The time-series prediction is based on the probabilistic weighted Kalman filter predictions. We make a probability interval about that estimate also based on the filter probabilities. In a Monte Carlo analysis, we test this MMAE approach and report the results based on many different criteria. Our analysis tests the robustness of the approach by testing its ability to make predictions when the Kalman filter dynamics models did not match the data generation time-series model. Our analysis indicates benefits in applying multiple model adaptive estimation for time series analysis.

MULTIPLE MODEL ADAPTIVE ESTIMATION FOR TIME SERIES ANALYSIS

CHAPTER I: INTRODUCTION

1.1 Overview

In this chapter we introduce the development of Multiple Model Adaptive Estimation (MMAE) [4, 5, 11, 13, 14, 16, 21, 22] for Time Series Analysis. In Section 1.2, we present the problem perspective and background for time series analysis. In Sections 1.3 and 1.4, we define the details of research objectives, motivation for using MMAE for time series analysis, scope of research, limitations and assumptions to accomplish this research. Section 1.5 presents the methodology. Finally Section 1.6 is the overview of following chapters.

1.2 Problem Perspective

A time series is set of observations on a quantitative variable collected sequentially in time [1, 3, 15]. Many sets of data appear as time series: a monthly amount of goods shipped from a factory, monthly aircraft spare part stock levels in an air force, weekly numbers of accidents on a road, the daily closing values of a stock market, hourly yields of a chemical reaction and many other examples. We can find examples of time series from many different fields: economics, engineering, natural or social sciences, military, business and so on.

In business, people are always interested in forecasting future values of a time series variable. They try to forecast the costs, sales, profits, inventory, backorders and etc. The military attempts to predict spare part inventories, aircraft mishaps, personnel requirements and many other things. To make accurate time series forecasts, we need time series analysis techniques.

Time series analysis is a method used for making predictions about future values of a time series variable based on past observations of that series and mathematical or statistical model.

Unlike the analysis of random samples of observations that are discussed in the context of most other statistics, the analysis of time series is based on the assumption that successive values in the data file represent consecutive, dependent measurements taken at equally spaced time intervals. The nature of this dependence of a time series is a major interest. Time series analysis concerns techniques for the analysis of this dependency. There are two main goals of time series analysis: (a) identify the pattern represented by the sequence of observations, and (b) forecast future values of the time series variable. Once the pattern is established, we can extrapolate the identified pattern to predict future events. The basic structure of time series models is:

DATA = PATTERN + NOISE [9]

PATTERN: Fundamental relationship describing the expected or average performance of the data over time. Implied "assumption of continuity" of pattern over forecast period.

NOISE: Randomness, "error," or variability around the pattern.

With this structure, the process of specifying a model to represent a time series often becomes one of pattern recognition.

1.3 Research Objectives

The time series analysis literature presents many analysis techniques: classical techniques such as moving averaging and smoothing methods, and Box-Jenkins ARMA, Autoregressive Moving Average, models are some of the most popular [1, 3, 9, 12, 15]. Classical techniques are generally appropriate for forecasting time series that exhibit stable patterns, which can be modeled as relatively straightforward functions of time. However, in real-life research and practice, patterns of the data are unclear, individual observations involve considerable error. Furthermore we still need to uncover the hidden patterns in the data and make good forecasts. The ARMA methodology developed by Box-Jenkins allows us to do just that.

We propose completely a new and different approach, MMAE, for time series estimation. We use ARMA models as our required dynamics model in MMAE because of its capability of representing almost all kinds of time series. Normally, the application of Box-Jenkins ARMA models requires to choose only one model which best fits the data. In our new method, MMAE, in order to forecast future values of a time series, instead of choosing one best ARMA model, we use all models - each model has different set of parameters- that represent time series adequately. We basically use the all most likely region of ARMA parameter space instead of limiting ourselves to only one set of parameters. MMAE allows us to run multiple models and the Kalman filter structure in MMAE weights each model, where less likely models receive less Bayesian probability.

The probabilistic weighted average of each Kalman filter's prediction gives the MMAE forecast. A significant advantage of our proposed approached is that it combines the Box-Jenkins steps of identification, and estimating, and automates it. Our primary objective is to test MMAE's ability to make time series predictions.

1.4 Scope of Research

Our research is basically limited to the application of MMAE to time series analysis.

MMAE is a Bayesian [9, 20, 21, 22] technique that applies bank of Kalman filters to predict future observations. In fact MMAE is nothing more than an algorithm that uses multiple filters in parallel to represent the dynamic nature of the system.

An overview of the MMAE algorithm follows [16]: The set-up for employing MMAE is to discretize the continuous space for each parameter into a set of representative points. The MMAE algorithm processes measurements (time series data values in this application) through a Kalman filter at each combination of discrete parameters. Each filter's residuals determine the probability of that filter's parameters being correct, conditioned on the measurements processed to that time. After processing all the available measurements, the filter probabilities indicate the likelihood of the parameters in that filter being correct conditioned on the measurements.

The Kalman filter is simply an optimal recursive, two-step data processing algorithm [6, 10, 12, 16, 23]. It propagates an estimate at the first step and then updates the estimate with an imprecise measurement, repeatedly. Each Kalman filter uses its own model to develop estimation of current state independent of the other filters. Unlike other

processing concepts, Kalman filter does not require all previous data to be kept in storage and reproduced every time a new measurement is taken. The Kalman filter is discussed in detail in Chapter II.

When we apply Box-Jenkins' method, we should estimate the parameters of the selected ARMA model, by some sort of estimation technique, like least square estimation or maximum likelihood estimation. However we may lose some important properties of another models that also represent the time series adequately, by not selecting them. We contend that if more than one model represents a time series adequately, they should each be retained. MMAE probabilistically weights each model, incorporates their predictions based on their associated probabilities and makes a forecast. It has two major advantages in this application to time series analysis. First, it combines the identification and estimation steps of traditional Box-Jenkins' method. The entire range of model parameters and model may be implemented, rather than selecting a single model in the Box-Jenkins' identification stage. Based on the data, the algorithm assigns probabilistic weights to each model. If two or more models appear to fit the data, MMAE uses all model estimates with appropriate probabilistic weights. Second, MMAE adjusts the model probabilistic weights for changes in the data pattern.

1.5 Methodology

We use Monte Carlo simulation to generate notional time series data, using an ARMA model, apply MMAE approach to this data, test this approach and report the results depending on many different criteria. The details and algorithm are given in Chapter III.

1.6 Thesis Overview

Many researchers have used MMAE for many different kinds of problems successfully [6, 8, 11, 13, 19, 21, 24]. Furthermore some researchers have obtained good forecasts using a single Kalman filter [10], so we test MMAE's ability to make time series predictions. Since we are unable to find any documented application of MMAE to time series analysis in the literature, we believe that this is a new and untried approach.

In the following chapters we develop this research in more detail. Chapter II gives the literature review of relevant subjects: Kalman filters, and Multiple Model Adaptive Estimation. Chapter III presents our research methodology by discussing the development of MMAE for time series analysis. Chapter IV gives the results from our analysis based on many different criteria. Finally Chapter V summarizes these results and concludes with our recommendations for future studies.

CHAPTER II: HISTORY AND BACKGROUND

2.1 Overview

In this chapter we present the background of Multiple Model Adaptive Estimation, and the Kalman filters. In section 2.2 we discussed history of MMAE and Kalman filters, then we continue with the overview of MMAE in Section 2.3, Kalman filter in Section 2.4, and the detailed MMAE algorithm in Section 2.5.

2.2 History

How to best make inferences from a time series realization of data is a problem with a long history. Much of the early work in time series, such as by Bernoulli, Gauss, and Legendre, grew out of problems in astronomy. The problem they addressed involved making inference as to the location of a "heavenly body" from a sequence of imperfect observations. Gauss and Legendre developed and applied least square estimation in 1795. R. A. Fisher introduced maximum likelihood estimation in 1912. [23]

In the 1800's, the Danish astronomer T. N. Thiele developed a recursive procedure resembling what is now referred to as the Kalman Filter for the problem of determining the distance from Copenhagen to Lund. Kolmogorov in 1941 and Wiener in 1942 independently developed a linear minimum mean-square estimation technique that received considerable attention and provided the foundation for subsequent development of Kalman Filter theory. [22, 23]

Kalman's and Gauss' work are significantly related. The Kalman filter can be rightfully regarded as an efficient computational solution of the weighted least square

method. The noise and initial state (presented in section 2.4) are essentially assumed by Kalman to be Gaussian (normally) distributed. Kalman assumes that the noise is independent from one sampling time to the next time; therefore he agrees with Gauss' assumption [23]. The basic assumptions of Gauss and Kalman are identical except that the latter allows the state to change from one time to the next.

Kalman published his first famous paper in 1960, and Kalman and Bucy published another paper in 1961 [23]. Almost immediately after Kalman's paper appeared (1960), a great interest from the aerospace community in application of procedures was aroused. Kalman met with the individuals at the NASA Ames Research Center in the fall of 1960 to discuss the potential application of his work in the navigation and guidance problem for a possible lunar mission. In August 1961, NASA extended the first major Apollo contract to the M.I.T. Instrument Laboratory to develop an on-board navigation and guidance system, which was to rely on state-space and Kalman filtering ideas [22].

State-space modeling and Kalman filtering is now the approach of choice for a wide range of time series problems [10, 12]. Their flexibility in handling multivariate data and nonstationary processes provides a significant advantage over traditional time series techniques for many applications, such as geophysical exploration, biomedicine, demography, nuclear power plant failure detection, and macroeconomics forecasting, in addition to traditional aerospace applications.

D. T. Magill [14] presented the MMAE for the first time in 1965. He arranged a number of Kalman Filters, each with different time invariant plant models and based on a hypothesized parameter realization, and used the residuals from these filters to form an

appropriately weighted sum of Kalman filter estimates. He showed that this adaptive estimation algorithm produced the optimal estimate in the minimum mean square error sense for a Gauss-Markov process. Although it was not specifically named MMAE, the structure of using multiple Kalman Filters was put into place.

Chang and Athans extended Magill's work to handle discrete systems [4, 5]. Many researchers have focused on exploiting the capability of MMAE to provide state or parameter estimation separately, as well as attempting to blend them.

Due to popularity of Kalman filters in the aerospace industry, USAF frequently used MMAE to solve specific Air Force problems. Dr. Peter Maybeck at Air Force Institute of Technology, AFIT, has supervised extensive research in the application of MMAE [8, 11, 13, 19, 21, 24].

2.3. Multiple Model Adaptive Estimation Review

In this section we present the overview of MMAE. Because the equations behind the MMAE are Kalman filter equations, the detailed algorithm is presented in Section 2.5 following the Kalman filter discussion.

MMAE is a Bayesian technique that applies a bank of parallel Kalman filters to predict future observations. Each Kalman filter is based on a different set of parameters and hence produces different residuals. The likelihood associated with each Kalman filter prediction is determined by the magnitude of residuals.

The basic structure of MMAE is shown in Figure 1. The major strength of MMAE is its ability to reconfigure rapidly in the presence of failures and thus provide accurate state estimates.

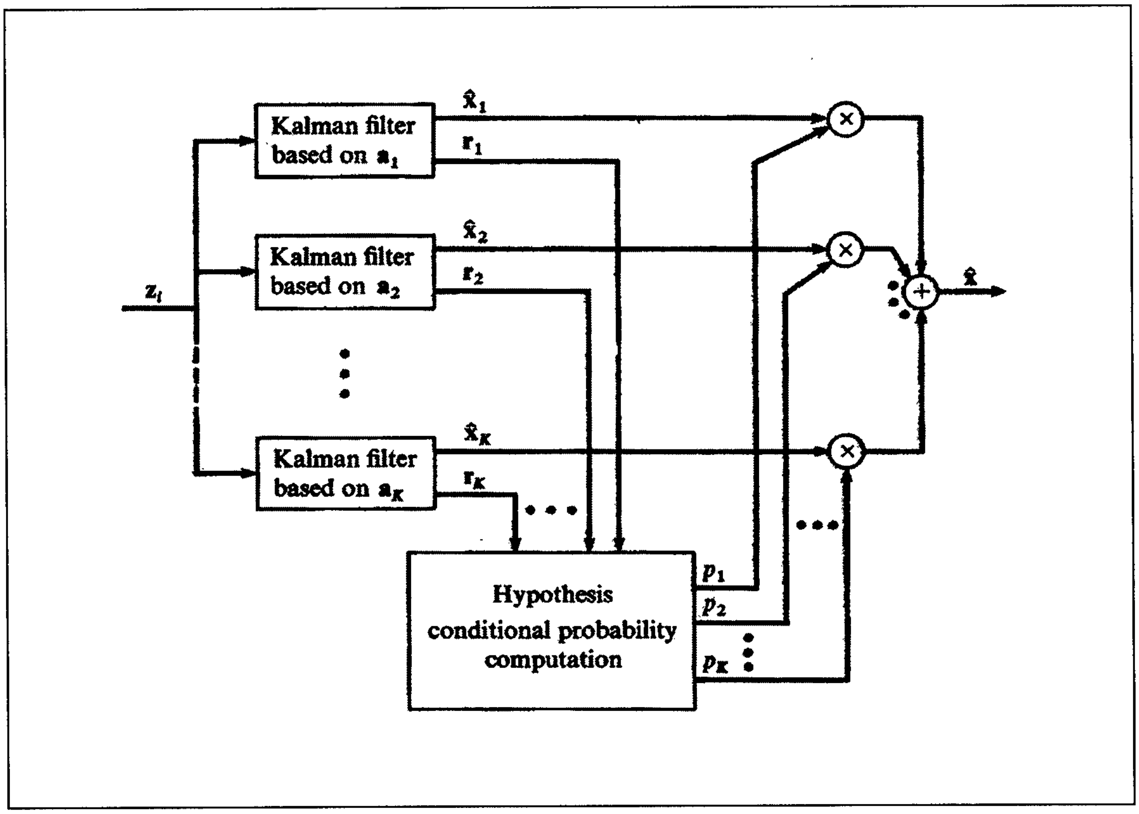

Figure 1. Multiple Model Adaptive Estimation Algorithm [17]

An overview of the MMAE algorithm follows: First the continuous space is discretized for each parameter yielding a set of representative points. The MMAE algorithm processes the measurements through a Kalman filter at each combination of these discrete parameters. Each filter's residuals determine the probability of that filter's parameters being correct, conditioned on the measurements processed to that time. After processing all the available measurements, the filter probabilities indicate the likelihood of the parameters in that filter being correct conditioned on the measurements.

This procedure is based on the following assumptions [24]:

- The sampled-data system is adequately represented by a linear stochastic state differential model for a given parameter vector value, resulting in Gaussian probability density functions, and can be described equivalently by linear stochastic differential equations [16, 17, 18]. In case nonlinear models are required to describe the system adequately, extended Kalman filters [6, 14, 22] replace the linear Kalman filter in the MMAE.
- The uncertain parameters to be estimated affect the system matrices or the statistics of the noises entering the system.
- MMAE theory assumes a discrete-valued parameter vector. Actually, a parameter value typically varies over a continuous range of the parameter space. Thus, parameter values have to be discretized to some level of resolution for feasible implementation. Clearly, poor choices in discrete values for a continuous parameter may result in poor modeling by the MMAE elemental filters and thus poor estimation from the entire MMAE itself. This results when an elemental filter within the MMAE bank does not have a good model of the system's current behaviour.

2.4 Kalman Filter Theory

In this section, we present the general Kalman filter theory applicable to continuous and time-varying systems. The time series, forecasting application will be limited to discrete, time-invariant Kalman filters in Chapter III. The notation is based on Maybeck's notations [16].

2.4.1 Introduction

A Kalman filter is an optimal, recursive, two-step data processing algorithm. It is optimal for any criterion that makes sense and is chosen to evaluate the performance of a system. The Kalman filter incorporates all available information, including all measurements regardless of their accuracy to estimate the state of a system. The state of the system refers to a set of variables that describe the inherent properties of the system at a specific instant in time. It is recursive, because the same two steps of propagation and update are repeated for each additional observation. Furthermore, it doesn't require all previous data be kept in storage and reprocessed every time a new measurement is taken. Filter or data processing algorithm refers to a computer program implements the mathematical algorithm.

The need for a filter is apparent from the Figure 2. Often times the state of system can't be measured directly, and measuring devices are used to infer these values. Unfortunately any measurement contains some degree of noise, bias and device inaccuracies, and a valuable estimate must be derived from those noisy signals.

A Kalman filter uses all indirect measurements of data, plus prior knowledge of system and covariance information of both the state variable and indirect measurements, to estimate the desired state variable in a manner that minimizes the error statistically.

From a Bayesian viewpoint [17, 20, 22], the Kalman filter propagates the conditional probability density of the desired variable, conditioned on actual data coming from measuring devices. The conditional probability density propagation depends on assumptions that linear models can represent the system and that the system and measurement noises are white and follow a Gaussian (Normal) distribution.

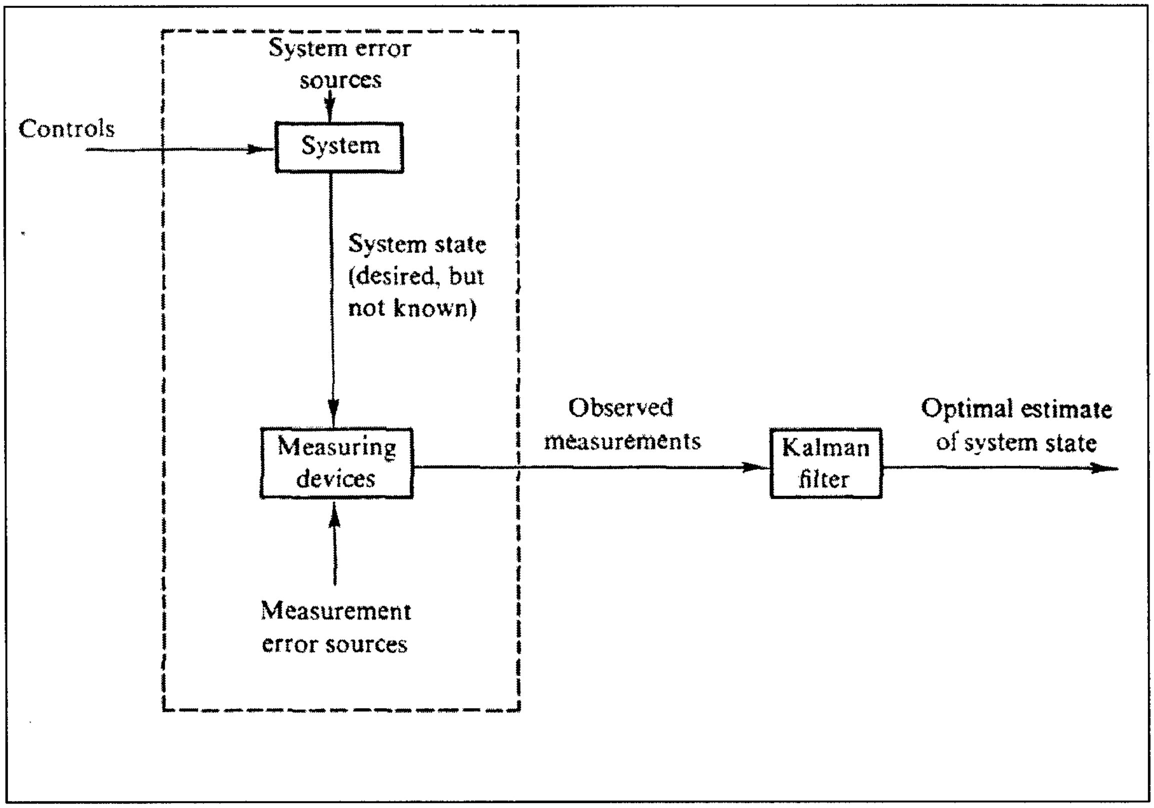

Figure 2. A Typical Kalman Filter Application [16:5]

2.4.2 System Models

To apply the Kalman filter, we generally need [8]:

1) A dynamic model that indicates how the state vector changes over time.

2) A measurement model that relates observations to the state vector and updates estimates based on these observations.

3) An estimate of the variance of the dynamic noise covariance.

4) An estimate of measurement (or observation) noise covariance.

In some applications we don't have estimates of both the dynamic and measurement noise covariances. For a series of scalar values, we cannot simultaneously

estimate both, since they are indeterminate. Instead we may directly estimate the ratio, which is the Kalman filter gain. The Kalman filter gain is presented in Section 2.4.3.

Before considering Kalman filter equations, we present underlying stochastic equations: linear stochastic difference equations, and associated dynamic models and measurement models. Since we use discrete, time-invariant Kalman filters, we limit the discussion to discrete-time system models in this section as well.

2.4.2.1 Linear Stochastic Difference Equation

The general form of the linear stochastic difference equation is

$$\mathrm{X}(t_n) = \Phi(t_{n,}t_{n-1})\mathrm{X}(t_{n-1}) + \mathrm{B}_d(t_{n-1})u(t_{n-1}) + G_d(t_{n-1})w_d(t_{n-1}) \tag{1}$$

where

t_n is the n^{th} time interval,

the indices $_d$ shows that they are discrete,

X is a vector of system states,

Φ is transition matrix of states from one time to next time,

B_d is discrete system control input matrix,

u is discrete deterministic control input matrix,

G_d is discrete dynamic noise input matrix,

w_d is zero-mean white Gaussian noise vector with covariance $Q_d(t_{n-1})$ [16].

The mean and covariance of the $\mathrm{X}(t_n)$ process defined by Equation 1 are

$$m_x(t_n) = \Phi(t_{n,}t_{n-1})m_x(t_{n-1}) + \mathrm{B}_d(t_{n-1})u(t_{n-1})$$

$$\mathrm{P}_{xx}(t_n) = \Phi(t_{n,}t_{n-1})\mathrm{P}_{xx}(t_{n-1})\Phi^T(t_{n,}t_{n-1}) + G_d(t_{n-1})Q_d(t_{n-1})G_d^{\ T}(t_{n-1})$$

2.4.2.2 Dynamics Model

The discrete-time version of the dynamics equation can be obtained by integrating the continuous-time dynamic equations over time. So the discrete-time dynamics equation without control inputs,

$$X(t_n) = \Phi(t_{n,} t_{n-1}) X(t_{n-1}) + G_d w_d(t_{n-1})$$

where

$$w_d(t_{n-1}) = \int_{t_{n-1}}^{t_n} \Phi(t_{n,} \tau) G(\tau) d\beta(\tau) \text{ , and}$$

β is dynamic driving noise, which can be modeled as Brownian motion [16].

The transition matrix Φ relates the state vector $X(t_{n-1})$ at one time to a state vector at the next time index $X(t_n)$. $w(t_{n-1})$ is normally distributed with mean of zero and have the properties:

$$E[w_d(t_{n-1})] = 0$$

$$E[w_d(t_{n-1}) w_d^T(t_{n-1})] = \int_{t_{n-1}}^{t_n} \Phi(t_{n,} \tau) G(\tau) Q(\tau) G^T(\tau) \Phi^T(t_{n,} \tau) d\tau = Q_d(t_{n-1}) \quad (2)$$

$$E[w_d(t_i) w_d^T(t_j)] = 0 \text{ for } t_i \neq t_j$$

Discrete-time dynamics equation can be also represented as

$$X(t_n) = \Phi(t_{n,} t_{n-1}) X(t_{n-1}) + G_d(t_{n-1}) w_d(t_{n-1}) \quad (3)$$

If we assume an equally spaced and time-invariant system model, the transition matrix is constant, that is $\Phi(t_{n-1.} t_n) = \Phi$. The dynamic noise $w_d(t_{n-1})$ is assumed to have constant variance, $Q_d(t_{n-1}) = Q_d$, and also noise input matrix $G(t_{n-1})$ is also assumed to be constant.

The time-invariant state dynamics model can be represented as

$$X(t_n) = \Phi X(t_{n-1}) + Gw(t_{n-1}) \tag{4}$$

We use this dynamic model to represent ARMA models later in Chapter 3.

2.4.2.3 Measurement Model

The modeled measurement or observation vectors, $Z(t_n)$, are linearly related to the state, but observed with measurement errors:

$$Z(t_n) = \mathbf{H}(t_n)\mathbf{X}(t_n) + v(t_n) \tag{5}$$

where

Z is the measurement or observation,

H is the measurement or observation matrix,

X is the vector of system states,

v is the measurement noise.

The measurement noise is assumed to be normally-distributed white noise with zero mean, such that

$$E[v\ (t_n)] = 0$$

$$E[v(t_n)v^T(t_n)] = R(t_n), \text{and}$$

$$E[v(t_i)v^T(t_j)] = 0 \text{ for } t_i \neq t_j$$

The covariance of the time-invariant measurement white noise is R. The dynamics noise and measurement noise are assumed to be uncorrelated. The measurement matrix $H(t_n)$ and measurement noise process $R(t_n)$ are assumed not to change with time. Therefore, they are replaced with H and R respectively. The time-invariant measurement model can be expressed as

$$Z(t_n) = \mathbf{H}\mathbf{X}(t_n) + v(t_n) \tag{7}$$

We employ this model assuming the ARMA data are observed with additive observation noise.

2.4.3 Discrete Space-Time-Invariant Kalman Filter Algorithm

In this section, we present the two stages of Kalman filtering: the propagation stage and the measurement update stage. First let us define some terms. Discrete means that two stages occur only at set intervals, as opposed to continuous. Time-invariant implies that the matrices Φ, G, and H do not change throughout the process (stationary process). The covariance matrices of the noise terms Q_d and R are constant, assuming the process is stationary.

At the time of an observation, two state estimates exist. These state estimates are indicated by a hat over the state vector. The estimate prior to a measurement update is labeled with a superscript minus sign, $\hat{\mathbf{X}}(t_n^-)$. Similarly, the state estimate after measurement update is labeled with a superscript plus sign, $\hat{\mathbf{X}}(t_n^+)$. The associated state covariance matrices are $\mathbf{P}(t_n^-)$ and $\mathbf{P}(t_n^+)$, respectively.

These state estimates are the conditional distribution of the state vector. With the Gaussian (normally distributed) assumption, the state estimates and associated covariance matrices specify the conditional distribution of the true state (in a Bayesian sense).

2.4.3.1 Propagation Stage

The Kalman filter iterates between propagations through time and updates for each available measurement. The Kalman filter gain weights the dynamic model versus the

measurement information when updating estimates. The state estimate, which is the mean of the conditional state distribution, propagates through time with

$$\hat{X}(t_n^-) = \Phi(t_n, t_{n-1})\hat{X}(t_{n-1}^+) + G_d w_d(t_{n-1}) \tag{8}$$

where

$E[w_d(t_{n-1})] = 0$,

$\hat{\mathbf{X}}$ is the estimated state vector,

Φ is the transition matrix,

G_d is the dynamic noise input matrix.

The associated growth in the covariance matrix determined by

$$P(t_n^-) = \Phi P(t^+{}_{n-1})\Phi^T + G_d Q_d G_d{}^T \tag{9}$$

where

P is covariance matrix of state estimates,

G_d is the dynamic noise, $w_d(t_n)$, input matrix, and

Q_d is the covariance matrix of discrete dynamic driving noise.

2.4.3.2 Measurement Update Stage

The measurement stage combines two sets of information: a state estimate based on the dynamics model in the propagation stage and a correction to this estimate based on an actual measurements and the measurement model. For known dynamics noise covariance matrix **Q** and known measurement noise matrix **R**, the state covariance matrices are independent of the actual observations. The Kalman filter provides weighting between the two sets of information about the state. Kalman filter gain is

$$\mathbf{K}(t_n) = \mathbf{P}(t_n^-)\mathbf{H}^T[\mathbf{H}\mathbf{P}(t_n^-)\mathbf{H}^T + \mathbf{R}]^{-1} \tag{10}$$

where

H is measurement or observation matrix.

After measurement at time t_n, the updated state estimate is

$$\hat{\mathbf{X}}(t_n^+) = \hat{\mathbf{X}}(t_n^-) + \mathbf{K}(t_n)[\mathbf{z}(t_n) - \mathbf{H}\hat{\mathbf{X}}(t_n^-)]$$

where

Z_n is actual measurement, and

$\mathbf{H}\hat{\mathbf{X}}(t_n^-)$ is the prediction of measurement based on assumed measurement model. If we define the residual vector as the measurement minus the expected measurement

$$\mathbf{r}(t_n) = \mathbf{z}(t_n) - \mathbf{H}\hat{\mathbf{X}}(t_n^-) \tag{11}$$

we can rewrite the update equation as

$$\hat{\mathbf{X}}(t_n^+) = \hat{\mathbf{X}}(t_n^-) + \mathbf{K}(t_n)\mathbf{r}(t_n) \tag{12}$$

The update of the covariance relationship is calculated with

$$\mathbf{P}(t_n^+) = \mathbf{P}(t_n^-) - \mathbf{K}(t_n)\mathbf{H}\mathbf{P}(t_n^-) \tag{13}$$

Again, for known dynamics noise covariance matrix **Q** and known measurement noise matrix **R**, the state covariance matrixes are independent of the actual observations.

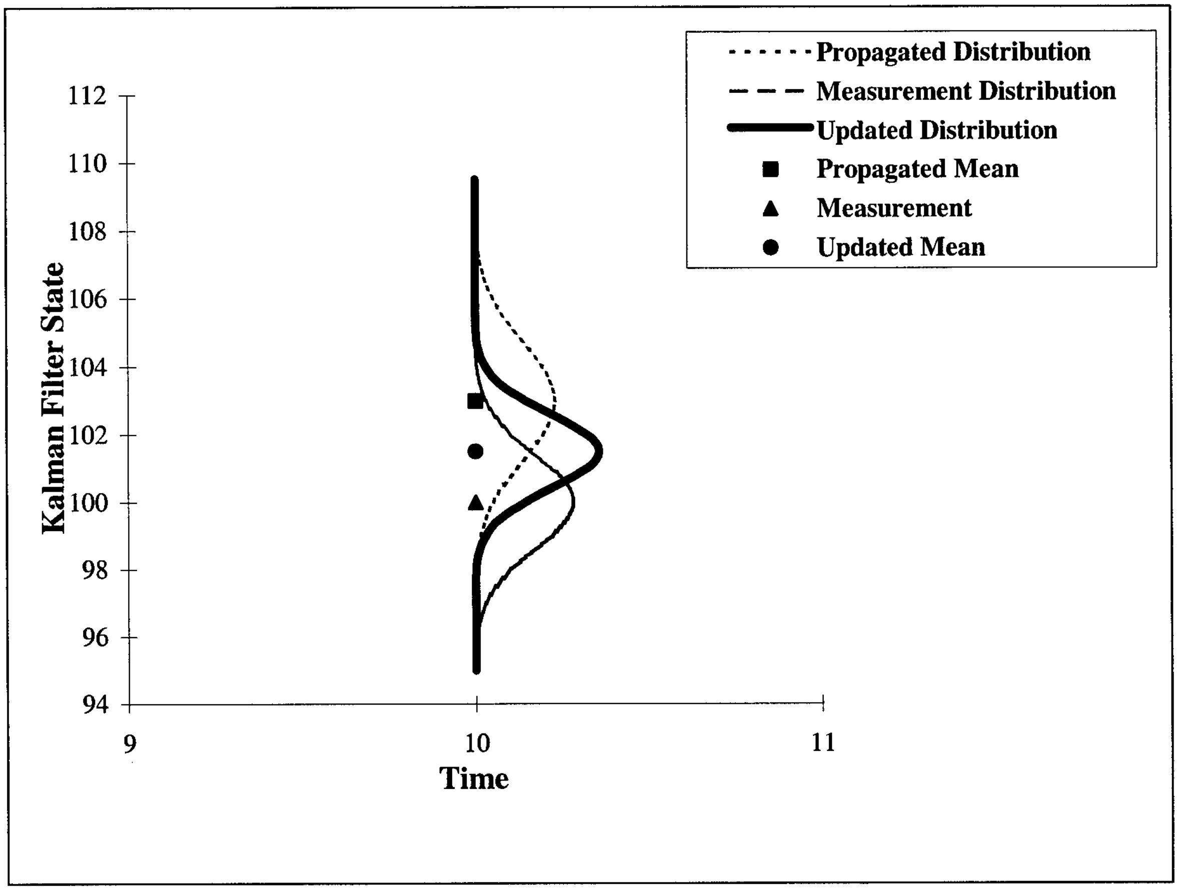

Figure 3. Kalman Filter Process [8]

In many engineering applications, these covariances matrices are pre-computed and stored. A single propagation and update stages of a Kalman filter process is illustrated in Figure 3.

2.5 Multiple Model Adaptive Estimation Algorithm

The theoretical development of MMAE is given in Section 2.1. In this section, we present the algorithms of MMAE. The theory and notation presented in this section closely follow the development by Maybeck [16, 17].

Let **a** denote a vector of parameters to be estimated (by MMAE), which affect any or all of Φ, B_d, H, Q_d, and R. The continuous range of values for **a** is discretized into K representative sets of values ($a_1, a_2, \ldots\ldots, a_k$), where each a_k corresponds to model with k^{th} Kalman filter. The Equations 1, 2, 5, and 6 with the k^{th} filter model can be given as

$$\mathrm{X}_k(t_n) = \Phi_k(t_n, t_{n-1})\hat{\mathrm{X}}_k(t_{n-1}) + G_{dk}(t_{n-1})w_{dk}(t_{n-1})$$

$$Z_k(t_n) = \mathbf{H}_k(t_n)\mathbf{X}_k(t_n) + v_k(t_n) \quad \text{with}$$

$$E[w_{dk}(t_{n-1})w_{dk}{}^T(t_{n-1})] = Q_{dk}(t_{n-1})$$

$$E[v_k(t_n)v_k{}^T(t_n)] = R_k(t_n)$$

Based on this model, the Kalman Filter propagation and measurement update Equations 8, 9, 10, 11, 12, and 13 are also given with addition of subscript k on all variables

$$\hat{\mathrm{X}}_k(t_n{}^-) = \Phi_k(t_n, t_{n-1})\mathrm{X}_k(t_{n-1}{}^+) + G_{dk}w_{dk}(t_{n-1})$$

$$\mathrm{P}_k(t_n{}^-) = \Phi_k\mathrm{P}_k(t^+{}_{n-1})\Phi_k{}^T + G_{dk}Q_{dk}G_{dk}{}^T$$

$$\mathbf{r}_k(t_n) = \mathbf{z}_k(t_n) - \mathbf{H}_k\hat{\mathbf{X}}_k(t_n^-)$$

$$\mathbf{K}_k(t_n) = \mathbf{P}_k(t_n^-)\mathbf{H}_k{}^T[\mathbf{H}_k\mathbf{P}_k(t_n^-)\mathbf{H}_k{}^T + \mathbf{R}_k]^{-1}$$

$$\mathrm{A}_k(t_n) = \mathbf{H}_k\mathbf{P}_k(t_n^-)\mathbf{H}_k{}^T + \mathbf{R}_k \quad \text{, which is variance matrix of measurements}$$

$$\hat{\mathbf{X}}_k(t_n^+) = \hat{\mathbf{X}}_k(t_n^-) + \mathbf{K}_k(t_n)\mathbf{r}_k(t_n)$$

$$\mathbf{P}_k(t_n^+) = \mathbf{P}_k(t_n^-) - \mathbf{K}_k(t_n)\mathbf{H}_k\mathbf{P}_k(t_n^-)$$

Each $\hat{X}_k(t_n^{\,+})$ is then calculated using a Kalman filter with the associated a_k parameters. The probabilities are calculated using their residuals $\mathbf{r}_k(t_n) = \mathbf{z}_k(t_n) - \mathbf{H}_k \hat{\mathbf{X}}_k(t_n^-)$ and their covariance matrices $\mathrm{A}_k(t_n) = \mathbf{H}_k \mathbf{P}_k(t_n^-) \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k$. Since these are jointly normally distributed, the likelihood of each filter is calculated by

$$f_{Z(t_n)|a,Z(t_{n-1})}(z_n | a_k, Z_{n-1}) = (2\pi)^{\frac{-S}{2}} |\mathrm{A}_k|^{\frac{-1}{2}} \exp\left\{ \frac{-1}{2} r_k^{\,T}(t_n) \mathrm{A}_k^{-1} r_k\,(t_n) \right\} \tag{14}$$

where S is the dimension of the measurement vector $\mathbf{Z}_n$, S=1 in this application.

Let the probability that **a** assumes the discrete value a_k, conditioned on the observed measurement history to time t_n be

$$p_k(t_n) = prob[a = a_k | \mathrm{Z}(t_n) = \mathrm{Z}_n]$$

Then $p_k(t_i)$ can be evaluated recursively for all k via the iteration

$$p_k(t_n) = \frac{f_{Z(t_n)|a,Z(t_{n-1})}(z_n | a_k, Z_{n-1}) p_k(t_{n-1})}{\sum_{k=1}^{K} f_{Z(t_n)|a,Z(t_{n-1})}(z_n | a_k, Z_{n-1}) p_k(t_{n-1})} \tag{15}$$

The largest likelihood, calculated in Equation 14, based on a residual in consonance with its covariance gives the best set of parameters with a high probability from Equation 15.

The probabilities at each measurement time, t_n, for n = 1,..., N, must sum to one:

$$\sum_{k=1}^{L} p_k(t_n \,|\, \mathbf{Z}_n) = 1$$

This normalization limits the conditional probabilities to only the K discrete parameter combinations used in the filters.

The Bayesian minimum mean squared error state estimate, the sum of the K discrete state estimates weighted by their associated probabilities, is the MMAE estimated state vector:

$$\hat{\mathrm{X}}_{MMAE}(t_n^{\;+}) = \sum_{k=1}^{K} \hat{X}_k(t_n^{\;+}) p_k(t_n) \tag{16}$$

The covariance matrix for the updated MMAE state vector $\hat{\mathrm{X}}_{MMAE}(t_n^{\;+})$ is

$$P_{MMAE}(t_n^{\;+}) = \sum_{k=1}^{K} p_k(t_n)\{P_k(t_n^{\;+}) + [\hat{X}_k(t_n^{\;+}) - \hat{X}_{MMAE}(t_n^{\;+})][\hat{X}_k(t_n^{\;+}) - \hat{X}_{MMAE}(t_n^{\;+})]^T\}$$

and the conditional mean for the unknown vector of system parameters **a** at time t_n is

$$\hat{a}(t_n) = E\{a - \hat{a}(t_n) \big| Z(t_n) = Z_n\} = \sum_{k=1}^{K} a_k p_k(t_n)$$

CHAPTER III: METHODOLOGY

3.1 Overview

Having already presented the detailed general MMAE algorithm in Section 2.5, this chapter presents its application to time series analysis. Since the MMAE algorithm is taken from engineering, and this is the first attempt to apply it to a time series, we explain it from an operational research point of view. We present state space models in Section 3.2 and Kalman filter Section 3.3. Finally Section 3.4 presents the MMAE forecasting technique

3.2. State Space Models

So far, we have assumed that the observed time series measurements are exact. Therefore, the classical and Box-Jenkins techniques assume that at time t the time series actually had the exact value of X_t. Box, Jenkins and Reinsel [3] note that we may employ models that assume superimposed or additive white noise. Equation 7 can express the corresponding system observation, where $\mathbf{X}_t$ is a multivariate time series with the components called state variables, H is the mapping matrix, and v is white noise.

We assume that state $\mathbf{X}_t$ depends on the previous state as in Equation 8, from which we delete the time indices of transition matrix, due to time invariance. This equation is called the state equation. Both the state equation and the measurement equation form a state space model. In time series perspective, $X(t_n)$ represents a time series that follows an ARMA structure. In the next section we show how to represent ARMA models in state space form.

3.3 Kalman Filters Method

Even if we use an ARMA dynamics model to represent the state of the system, X_t, the presence of observation noise induces two major changes to the forecasting process. The first change results from a lack of precise knowledge regarding the state of the system at a given time. We discuss the probability distribution of the true unobserved state. The second change, we must choose how to treat this true, but unknown, state. We take a Bayesian approach (imbedded in Kalman Filter) that this unknown state is a random variable. Therefore, we begin with a prior distribution of the state, and update this distribution to a posterior distribution based on the information provided by each observation.

The Kalman filter recursive equations enable easy calculation of one-step forecasts, provided a model is written in state space form. The details of Kalman Filter equations are given in Chapter II. In this chapter, we give the corresponding matrices and apply them to time series data.

3.3.1 The State Dynamics Model

The state dynamics model in this research is ARMA (2,2) model. We give the state space formulation of an ARMA (2,2) model below. The general form of mean-corrected ARMA (2,2) model is normally written as

$$\tilde{X}_t = \phi_1 \tilde{X}_{t-1} + \phi \tilde{X}_{t-2} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

where $\tilde{X}_t = X_t - \overline{X}$, with $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then we can write ARMA (2,2) model in state space form, $X(t_n) = \Phi X(t_{n-1}) + Gw(t_{n-1})$, with

$$\mathbf{X}(t_n) = \begin{bmatrix} \tilde{X}_t \\ \tilde{X}_{t-1} \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix}, \quad \Phi = \begin{bmatrix} \phi_1 & \phi_2 & -\theta_1 & -\theta_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{w}(t_{n-1}) = \varepsilon_t$$

where $\mathbf{w}(t_{n-1})$ is normally distributed with zero mean and variance of dynamic noise $\mathbf{Q} = \sigma_\varepsilon^2$.

3.3.2 The Measurement Model

The measurement or observation vector, $\mathbf{Z}(t_n)$, Equation 7, is linearly related to the state, but observed with measurement noise, where the time-invariant measurement white noise, $v(t_n)$, is normally distributed with zero mean and covariance **R**. The dynamic noise, Q, and measurement noise, R, are assumed to be uncorrelated. In our application each noises and their variances are scalars.

If the time series is the first element in the state vector of dimension m, the measurement vector **H** has m terms, [1, 0, 0, …,0] and the measurement noise is a single value. In our application m is equal to 4.

When the measurement model is used with an ARMA dynamics model, the modeled process is more general than a traditional ARMA. It is in fact an ARMA time series observed with measurement noise. The dynamics noise affects future observations, while the measurement noise only affects the observation at that time. In fitting an ARMA observed with measurement noise, the data series are used as the realized observations, Z_n.

3.3.3 Kalman Filter Forecasting

In applying a single Kalman filter, we go through the same four steps as the Box-Jenkins methodology: identification, estimation, diagnostics, and forecasting.

3.3.3.1 Model Identification

For ARMA dynamics models, model identification is the same as the Box-Jenkins methodology. The details exceed the scope of this thesis, but basically model identification is done by looking at the patterns of both autocorrelation and partial autocorrelation functions of data. As mentioned before, we accept ARMA (2,2) as our dynamics model [2,15]. In fact, Box-Jenkins [3] and Makridas et.al. [15] do not suggest using more parameterized model than an ARIMA (2,2,2), which covers a tremendous range of practical forecasting situations. Moreover the parsimony principle [3] tells us to use the minimum possible amount of parameters.

The selected ARMA (2,2) model is used for the dynamics equation in the Kalman filter. Since parameters may be estimated to be zero, the degenerative ARMA models are also possible.

3.3.3.2 Model Estimation

As in the ARIMA estimation, we conduct a nonlinear search of the parameter space. The Kalman filter parameter space has one more parameter than the underlying Box-Jenkins ARMA model. As in the ARMA model, we have ϕ_1, ϕ_2, …, ϕ_p, θ_1, θ_2, …, θ_q, as appropriate, and $\mathbf{Q} = \sigma_\varepsilon^2$ (dynamic noise variance in Box Jenkins and Kalman

filter notation). The additional parameter is either the first term of the Kalman filter gain, k, or the measurement noise variance R.

We may determine the Kalman filter gain using the set of ARMA coefficients to determine the transition matrix Φ along with the dynamic noise variance Q and the measurement noise variance R. Begin with any initial estimate of the $\mathbf{P}(t_0)$, we may iterate through the propagation and update equations, Equations 9, 10, and 13, until the Kalman filter gain becomes constant.

Notice that the methodology depends upon both the dynamics noise shock variance Q and the measurement noise variance R. While each noise is assumed to be independent, their impact on the Kalman filter gain is highly correlated. Maybeck [21] states that ratio of the magnitude of the eigenvalues determines the magnitude of the Kalman filter gain. In our application, since the noise and their variances are scalars, the ratio of Q and R (for any given set of model coefficients) determines the steady state Kalman filter gain, which is simply $K = \dfrac{Q}{Q+R}$. Therefore, we fix Q=1 and treat R as a multiple of Q. The true Q may be estimated from the residuals, if desired.

We apply the ARMA dynamics model by iteratively propagating and updating for all the available observations. We propagate the state estimates using Equation 8. We update the state based on the next observation and a steady-state gain with Equations 11 and 12. The residuals (equivalently errors) are used to evaluate each set of coefficients.

We conduct a nonlinear search to determine the model coefficients and R. The set of model coefficients, which maximize the maximum likelihood function and

equivalently the conditional joint distribution of observations, shown in Equation 17 [3:276-277], is selected as the best fitting model.

$$p(\mathbf{z}\mid \boldsymbol{\varphi},\boldsymbol{\theta},Q)=\prod_{i=1}^{i=n}\left\{\left(2\pi P_{11}^{-}(t_i)\right)^{-1/2}\exp\left[\frac{-\left(z_i-\hat{X}_1^{-}(t_i)\right)^2}{2P_{11}^{-}(t_i)}\right]\right\}$$
$$=\prod_{i=1}^{i=n}\left(2\pi P_{11}^{-}(t_i)\right)^{-1/2}\exp\left[\frac{-1}{2P_{11}^{-}(t_i)}\sum_{i=1}^{i=n}\left(z_i-\hat{X}_1^{-}(t_i)\right)^2\right] \tag{17}$$

We calculate the residuals with Kalman filter equations. Since only the ratio of Q and R is important, we take Q = 1 (arbitrarily, we can take any value) and R any starting value greater than zero.

During the nonlinear search phase, to eliminate the initialization bias, we first find the initial state estimates by backward forecasting or backforecasting. To begin the Kalman filter recursion for backforecasting and forecasting we need an initial state estimate and an initial covariance matrix. We use the expected mean corrected state, which is the zero vector, as the initial state estimate. We may select $P(t_0)$ to be very large, to represent a non-informative prior. If we desire the exact maximum likelihood estimates (MLE), for an ARMA (2,2) we fix R=0, which treats the measurements as exact, and solve for the expected covariance based on our state definition. We may use the general linear filter representation to determine this covariance matrix. Recall the general linear filter is

$$\tilde{X}_t = X_t - \mu = \sum_{k=0}^{\infty} \psi_k\, \varepsilon_{t-k}$$

Since the errors are uncorrelated

$$\mathrm{Var}(\tilde{X}_t) = \sum_{k=0}^{\infty} \psi_k^2\, \mathrm{Var}(\varepsilon_{t-k}) = \sigma^2 \sum_{k=0}^{\infty} \psi_k^2 \text{ , and}$$

$$\mathrm{Cov}(\tilde{X}_t,\ \tilde{X}_{t+k}) = \sigma^2 \sum_{i=0}^{\infty} \psi_i \psi_{i+k}$$

For an AR(2) process, the weights may be calculated with the recursive equation $\psi_j = \phi_1 \psi_{j-1} + \phi_2 \psi_{j-2}$ starting with $\psi_0 = 1$ and $\psi_{-1} = 0$. The coefficient of the moving average terms may be added directly to the appropriate filter coefficient later.

After calculating all weights for an ARMA(2,2) model, with the above recursion, the first two weights are recalculated with $\psi_1 = \phi_1 + \theta_1$ and $\psi_2 = \phi_1 \psi_1 + \phi_2 + \theta_2$. Thus, we may determine the initial state covariance matrix as, for an ARMA (2,2) dynamics model with state definition of

$$\mathbf{X}(t_n) = \begin{bmatrix} \tilde{X}_t \\ \tilde{X}_{t-1} \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix}$$

with

$$P_0 = \begin{bmatrix} Var\{\tilde{X}_t\} & \mathrm{cov}\{\tilde{X}_t, \tilde{X}_{t-1}\} & \mathrm{cov}\{\tilde{X}_t, \varepsilon_t\} & \mathrm{cov}\{\tilde{X}_t, \varepsilon_{t-1}\} \\ \mathrm{cov}\{\tilde{X}_t, \tilde{X}_{t-1}\} & Var\{\tilde{X}_{t-1}\} & \mathrm{cov}\{\tilde{X}_{t-1}, \varepsilon_t\} & \mathrm{cov}\{\tilde{X}_{t-1}, \varepsilon_{t-1}\} \\ \mathrm{cov}\{\tilde{X}_t, \varepsilon_t\} & \mathrm{cov}\{\tilde{X}_{t-1}, \varepsilon_t\} & Var\{\varepsilon_t\} & \mathrm{cov}\{\varepsilon_t, \varepsilon_{t-1}\} \\ \mathrm{cov}\{\tilde{X}_t, \varepsilon_{t-1}\} & \mathrm{cov}\{\tilde{X}_{t-1}, \varepsilon_{t-1}\} & \mathrm{cov}\{\varepsilon_t, \varepsilon_{t-1}\} & Var\{\varepsilon_{t-1}\} \end{bmatrix}$$

$$= \begin{bmatrix} Var\{\tilde{X}_t\} & \mathrm{cov}\{\tilde{X}_t, \tilde{X}_{t-1}\} & Q & \psi_1^2 Q \\ \mathrm{cov}\{\tilde{X}_t, \tilde{X}_{t-1}\} & Var\{\tilde{X}_{t-1}\} & 0 & \psi_1^2 Q \\ Q & 0 & Q & 0 \\ \psi_1^2 Q & \psi_1^2 Q & 0 & Q \end{bmatrix}$$

$$
= \begin{bmatrix} \sum_{i=0}^{\infty} \psi_i^{\ 2} & \sum_{i=0}^{\infty} \psi_i \psi_{i-1} & 1 & \psi_1^{\ 2} \\ \sum_{i=0}^{\infty} \psi_i \psi_{i-1} & \sum_{i=1}^{\infty} \psi_i^{\ 2} & 0 & \psi_1^{\ 2} \\ 1 & 0 & 1 & 0 \\ \psi_1^{\ 2} & \psi_1^{\ 2} & 0 & 1 \end{bmatrix} \tag{18}
$$

Therefore, with the Kalman filter equations, we may obtain the MLE values for ARMA models [10].

3.3.3.3 Forecasting

After the model parameters are selected, beginning with initial state estimates equal to zero vector, and an initial covariance matrix $P(t_0)$ as calculated in Equation 18, we process the available data through the Kalman filter using recursive equations. When we complete the last datum, the last state estimate is conditioned on all prior observations. Therefore future forecasts are all conditional estimates.

The one-step ahead forecast is available directly from the propagation equation:

$$\hat{\mathbf{X}}(t_{n+1}^{-}) = \Phi \hat{\mathbf{X}}(t_n^{+})$$

Predictions, l-steps into the future, require propagation of these estimates, without the update stage with

$$\hat{\mathbf{X}}(t_{n+l}^{-} \mid \mathbf{Z}(t_n) = \mathbf{Z}_n) = \Phi^{l} \hat{\mathbf{X}}(t_n^{+})$$

All Kalman filter state estimates are conditional. In the prior equation, the conditioning is explicitly included since it does not follow the convention of conditioning all observations prior to the time of the state estimate.

While missing data is difficult to analyze with classical and traditional Box-Jenkins techniques, the Kalman filter can easily handle missing observations. The best

state estimate is used in place of the missing observations. Therefore, we propagate without an update for missing observations.

3.3.3.4 Prediction Intervals

Based on the estimated dynamics noise variance and the measurement noise variance (or equivalently the Kalman filter gain), we may calculate all Kalman filter state covariance matrices. If we assume the states follow a Normal distribution, we may make a Bayesian probability interval for future forecasts. The mean is the state estimate based on the data available and the variance is the corresponding covariance matrix.

We use empirical prediction errors for the available data set to estimate prediction variance. We process the data through Kalman filter once more, this time making 1-step, 3-step and 5-step ahead forecasts within the data. All prediction errors are then collected and the MSE (mean square errors) of forecasts found for every step. Using these MSE's and standard prediction interval equation we find the prediction intervals.

3.4 Multiple Model Adaptive Estimation Forecast

The detailed algorithm is given in Chapter 2. In this chapter we present the time series forecasting application of MMAE.

MMAE applies a bank of Kalman filters to process the data simultaneously. Its major advantages are: first it combines the identification and estimation step, second it probabilistically weights the models, and gives more probability to the more likely model.

We begin MMAE by selecting discrete set of coefficients that cover the feasible parameter region, rather than searching for a single set of parameters that best fits the

data. For the AR (2) and MA (2) portions of ARMA (2,2) dynamics model, the coefficients are selected every 0.1 interval for ϕ_1, ϕ_2, θ_1, and θ_2 in their invertible triangle, Figure 4. The bank of Kalman filters consists of a single Kalman filter at each combination of these parameters.

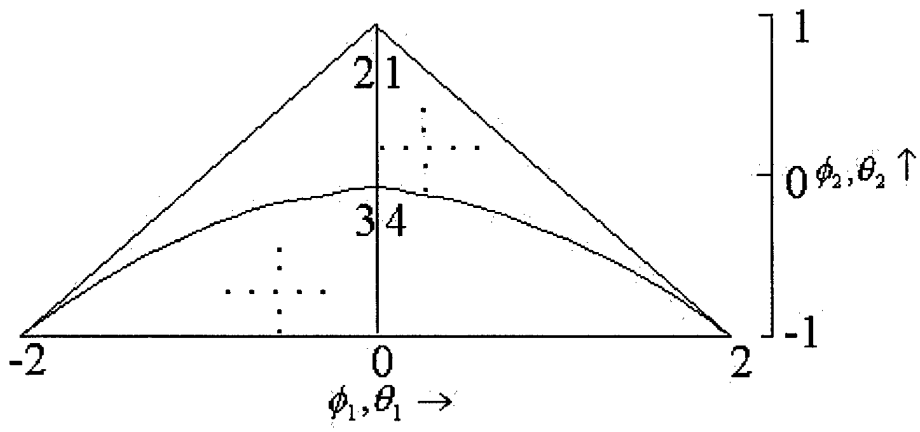

Figure 4. Parameter Selection from Invertible Triangle for AR and MA Models

Each Kalman filter processes data independently of each other, as discussed previously. Each filter starts conditionally with an initial state estimate of zero vector, and initial covariance vector P (t_0). All required equations are given in Chapter 2.

The MMAE algorithm calculates the probabilities associated with each of the K Kalman filters. Based on the assumption of zero mean and the estimated residual variance, the normal probability density function for the n^{th} measurement, z_n, conditioned on the k^h filter's vector of parameters, $\mathbf{a}_k$ and the prior measurement history, $\mathbf{Z}_{n-1}$, is calculated with Equation 14.

The probability for the k^{th} filter having the "correct" parameters conditioned on the measurement history through time t_n is calculated with Equation 15.

The initial or *a priori* probabilities account for information available about the likelihood of particular filter combinations before the measurement data are processed. If no information is available, the *a priori* probabilities should all be equal; $p_k(t_0) = 1/K$ for $k = 1,..., K$. In addition, if any of the filter probabilities become zero, that filter's probabilities remain zero. To prevent prematurely discarding potentially viable filter parameters, practitioners commonly apply a heuristic; if any of the filter probabilities decreases below a very small lower bound, such as 0.0001, the heuristic artificially increases that filter's probability to the lower bound. The filter probabilities that result after the last datum are not adjusted with this heuristic. The final filter probabilities represent the likelihood of each combination of model parameters conditioned on the available measurement history, $\mathbf{Z}_N$.

After determining the probabilities based on all available observations, we have a conditional (Bayesian) probability of each filter's parameters being correct. We use these probabilities to calculate a conditional estimate by probabilistically weighting each filter's forecast. The mean estimate of next-step forecast conditioned on the available measurement history, $\mathbf{Z}_N$, is calculated with Equation 16.

All l-step ahead forecasts for each filter can be made the same way the single Kalman filter does, where for each filter,

$$\hat{\mathbf{X}}_k(t_{n+l}^- \mid \mathbf{Z}(t_n) = \mathbf{Z}_n) = \Phi^l \hat{\mathbf{X}}(t_n^+)$$

Using the latest calculated filter probabilities we can weight each l-step forecast and find MMAE forecasts using

$$\hat{X}_{MMAE}(t_{n+l}{}^{+}) = \sum_{k=1}^{K} \hat{X}_k(t_{n+l}{}^{-}) p_k(t_n)$$

Using covariance propagation equation without the update stage, we calculate all variances for each *l*-step forecast of each Kalman filter as

$$P_k(t_{n+l}{}^{-}) = \Phi P_k(t^{+}{}_{n+l-1})\Phi^T + GQ\,G^T$$

Since we stop calculating covariance matrices after the Kalman filter gain becomes steady, we use the last calculated $P(t^{+}{}_n)$ of each filter for the first step covariances in the above equations.

Based on calculated covariances we find the prediction interval of every single forecast for every filter, then weight them with the filter probabilities for every step. The weighted sum of the lower bound and weighted sum of the upper bounds of prediction intervals give the final prediction intervals for MMAE forecasts.

CHAPTER IV: ANALYSIS AND RESULTS

4.1 Overview

In previous chapters we proposed a new method, MMAE, for time series forecasting. Chapter III gives the detailed algorithm of MMAE. We claim that MMAE is at least as good as other well-known classical and Box-Jenkins models in literature. In this chapter we conduct a Monte Carlo simulation to support our claim, and compare the results of MMAE with the other known forecast methods' results. Section 4.2 presents the implementation of the Monte Carlo simulation, Section 4.3 shows how we evaluate our method, Section 4.4 presents which statistical measures we use to check the model accuracies and interprets simulation outputs, and finally Chapter 4.5 gives the results and findings.

4.2 Design of Monte Carlo Simulation

The design of Monte Carlo study should be carefully optimized to cover all parameters, variables or criteria affecting the model. In our research there are four main areas of concern: sample size, noise variances, data generation parameters and techniques, and simulation trials, which are to be chosen carefully for inferential purposes.

As Box-Jenkins [3] suggest the sample size at least 100 to get satisfactory forecasts, we select sample sizes to be at least equal to or more than 100. Moreover we try 8 different sample sizes with the same parameters and same data, to measure the effect of sample sizes in MMAE forecasting. The simulation results suggest no affect of

sample size in MMAE's forecasting. We use different sample sizes, usually between 200 and 400. We become consistent, and use the same sample sizes for the same data generation parameters with different variances.

To measure the performance of MMAE against data fluctuations, we generate data with two different variances, big and small. The data are normally generated with the ARMA(2,2) equation with additive noise variance, $v(t_n)$,

$$X_t = \mu' + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} + v_t$$

where dynamic noise, $w(t_n)$, from Equation 4, and measurement noise, $v(t_n)$, from Equation 7 are distributed normally with mean and variances as shown in Table 1.

Table 1. Dynamic and Measurement Noise Variances

Dynamic Noise	Measurement Noise
$w(t_n) = \varepsilon_t \sim N(0,1)$	$v(t_n) \sim N(0,0.1)$
$w(t_n) = \varepsilon_t \sim N(0,25)$,	$v(t_n) \sim N(0,1)$

Data generation parameters are chosen to provide representation throughout the stationarity and invertibility regions. We test our method by generating many different data sets using different combinations of AR and MA parameters from their parameter regions, the invertible triangle, as shown in Figure 4 and Figure 5. We also generate data with pure autoregressive models, AR(1) and AR(2), pure moving average models, MA(1) and MA(2), and more parameterized mixed model, ARMA(3,3), to test the robustness of

MMAE. The number of cases, time series, from each data generation technique is given in Table 2. Data generation parameters and related simulation results are given in Appendix A and B.

Table 2. Number of Cases for Each Data Generation Model

ARMA(2,2)	AR(1)	AR(2)	MA(1)	MA(2)	ARMA(3,3)	Total
36	10	10	7	7	6	76

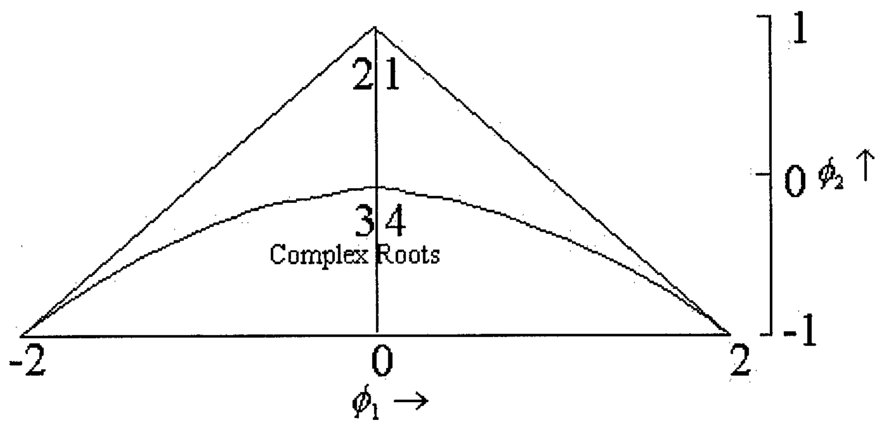

Figure 5. Typical Stationary and Invertible Parameter Space for AR(2) Models [3:61]

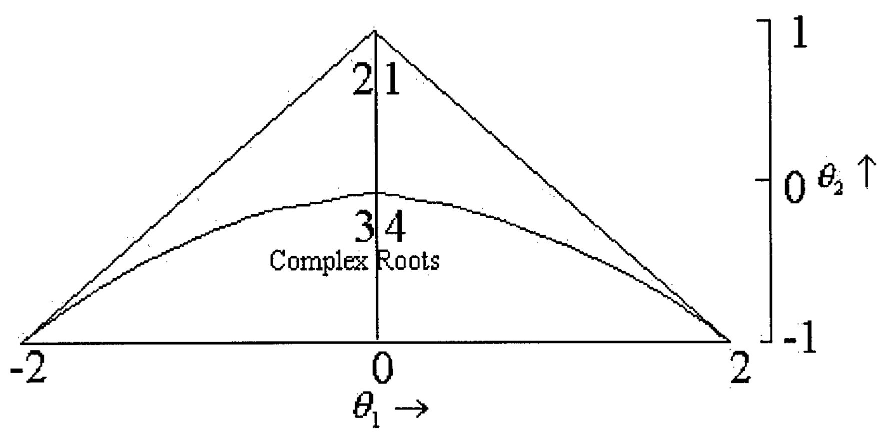

Figure 6. Typical Stationary and Invertible Parameter Space for MA(2) Models [3:76]

We split the data set into two parts, an initialization set and a test set or hold out set. Most of the data are used as initialization set to estimate any parameters and/or initialize the method. Forecasts are made for the test set. Since the test set is not used in model fitting, these forecasts are genuine forecasts made without using the values of the observations for these times. All statistics are computed for the errors in the test set only.

We run the simulation 1000 times for each parameter set, in order to get accurate results. The simulation results tables are the averages of a 1000 replications. Since we have 76 cases, our evaluation consists of 76,000 data series.

4.3 Evaluation Criteria

The evaluation our method is another important aspect of the research. We can simply fit MMAE to data, make forecasts, collect the statistics, and make inferences from those statistics. However none of these statistical measures give a good basis of comparison. Does an MSE of 6 or SSE of 56 indicate a good or bad forecasting performance?Another basis is to compare MMAE results with the other well-known sophisticated methods' results. The relative comparison of results obtained from MMAE and other methods is more meaningful and useful than simply computing statistics of the MMAE. So we decide to fit other methods as well as MMAE to data. The other forecast methods compared with MMAE are:

- Naïve
- Moving Average with window size of 5
- Moving Average with window size of 10
- Simple Average
- Exponential Smoothing
- Regression
- AR (1) ULS, unconditional least square model
- AR (2) ULS model

- MA (1) ULS model
- MA (2) ULS model
- ARMA (1,1) ULS model
- ARMA (2,2) model with 'TRUE' data generating parameters,
- ARMA (2,2) model with 'Initial Parameters Estimates' according to Box-Jenkins [3:202]
- ARMA (2,2) MLE, maximum likelihood mode
- Single Kalman Filter with dynamic model of ARMA (2,2)

Naïve method simply takes the most recent observation as forecast. Moving average methods take the average of 5 and 10 most recent observations as forecast. Simple average is the average of all previous observations. Exponential smoothing is the weighted average of the estimate made at previous time and the observation at this time. Regression uses the standard regression methods, by regressing the observations to time. Other methods except Kalman filter method are standard Box-Jenkins models. Further details about the methods can be found in [3, 7, 15]. ULS shows that all necessary parameters are estimated by unconditional least square method. MLE shows the parameters are estimated by maximum likelihood estimates. The parameters for the single Kalman filter method are also calculated using the maximum likelihood estimation procedure [10].

In this research we compared MMAE estimates with many other methods' estimates, but formally we take ARMA (2,2) MLE as standard as proposed by time series experts [3,8, 10, 15]. We give the analysis of results in the Section 4.5 based on this comparison.

4.4 Measuring Forecast Accuracy

In this section we give the brief description of statistics calculated to compare all methods.

All of the statistics are based on *k*-step-ahead forecast error (made at time *t*), which is

$$e_{t+k}(t) = \mathrm{X}_{t+k} - F_{t+k}(t)$$

where *k* represents all future times, *k*=1 to *n*,

X_{t+k} is *k*-step ahead observation,

F_{t+k} is *k*-step ahead forecast.

The statistical measures we use in comparison are: mean error (ME), mean absolute error (MAE), sum of square errors (SSE), 1-step prediction interval and its coverage (PIW1MSE and PIC1MSE), 3-step prediction interval and its coverage (PIW3MSE and PIC3MSE), and 5-step prediction interval and its coverage (PIW5MSE and PIC5MSE). The equations for these statistics for a single series with n forecast are given as

$$\mathrm{ME} = \frac{1}{n}\sum_{k=1}^{n} e_{t+k}$$

$$\mathrm{MAE} = \frac{1}{n}\sum_{k=1}^{n} \left|e_{t+k}\right|$$

$$\mathrm{SSE} = \frac{1}{n}\sum_{k=1}^{n} e^{2}{}_{t+k}$$

We use n=5 in our Monte Carlo analysis.

The 90% prediction intervals are computed as

$$F_{t+k}(t) \pm z_{\frac{\alpha}{2}}\hat{\sigma}_{k}$$

where $\hat{\sigma}_k = Var[e_{t+k}(t)]$ for all t. The prediction interval computation for MMAE is given in Chapter III. Prediction interval coverage is simply the number of times the prediction intervals contain real data across 5 forecasts and all replications.

Table 3 shows an example of a Monte Carlo simulation results; where true parameters are the parameters used to generate data; s*ample size* is the number of generated data; n*umber of predictions* is the number of test data set and forecasts; n*oise std. deviation* is the standard deviation of randomly generated dynamic noise; e*rror std. deviation* is the standard deviation of randomly generated measurement noise, which is added to data. The percentages at the bottom of table give the percentages of replications that MMAE estimates are better than ARMA (2,2) MLE estimates.

All the results in Table 3 and other tables in appendices are averages of 1000 replication for that data generation parameters set. In this case, MMAE's forecasts are better and more accurate than MLE's forecasts 53.5% and 55.5% of time out of the 1000 replications, with respect to statistics MAE and SSE. MMAE gives the tightest prediction intervals with high coverages. The narrow prediction interval express the forecast precision, the narrower the prediction interval, the more accurate the forecasts are. Especially the MMAE's prediction intervals are narrower than MLE's prediction intervals. Further conclusions are given in the next section.

Table 3. Monte Carlo Simulation Results

TRUE PARAMETERS

Phi1	=	-0.1		Sample Size	=	800
Phi2	=	0.1		Number of Predictions	=	5
Theta1	=	-0.6		Noise Std. Deviation	=	1
Theta2	=	-0.4		Error Std. Deviation	=	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.95	7.21	1.65	0.91	1.98	0.87	1.98	0.91
ARMA(2,2)InitialEstimates	-0.02	1.07	9.31	2.63	0.92	2.07	0.89	1.99	0.94
Naive	0.02	1.30	13.14	2.00	0.87	2.78	0.89	2.81	0.90
Moving Average (T=5)	0.03	1.06	9.17	2.08	0.92	2.37	0.88	2.38	0.88
Moving Average (T=10)	0.02	1.03	8.46	2.06	0.93	2.21	0.91	2.22	0.93
Simple Average	-0.01	0.98	7.59	1.99	0.91	2.00	0.87	2.00	0.91
Exponential Smoothing	0.01	1.21	11.55	1.87	0.91	2.47	0.85	2.50	0.89
Regression	-0.01	0.97	7.55	1.98	0.89	1.98	0.87	1.98	0.92
AR(1)ULS	-0.01	0.96	7.36	1.80	0.88	2.03	0.87	2.04	0.91
AR(2)ULS	-0.01	0.95	7.37	1.71	0.90	2.20	0.90	2.22	0.94
MA(1)ULS	-0.01	0.96	7.43	1.83	0.88	1.98	0.86	1.98	0.91
MA(2)ULS	-0.01	0.95	7.29	1.65	0.91	1.98	0.86	1.98	0.91
ARMA(1,1)ULS	-0.01	0.97	7.52	1.94	0.88	1.98	0.87	1.98	0.91
ARMA(2,2) ULS	-0.01	0.95	7.21	1.65	0.91	1.98	0.87	1.98	0.91
ARMA(2,2) MLE	-0.01	1.01	8.44	2.00	0.93	2.12	0.86	2.01	0.90
Kalman Filter ARMA(2,2)	-0.04	1.00	8.23	1.92	0.92	2.09	0.87	2.02	0.89
MMAE ARMA(2,2)	0.00	0.97	7.45	1.87	0.93	1.87	0.86	1.87	0.89

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	53.5	55.5

4.5 Results and Findings

All Monte Carlo Simulation Results are presented in Appendix B.

In an application of MMAE for a time series we recommend using all possible Kalman filters within the parameter space of AR and MA parameters in their invertible triangles as shown in Figure 4 and Figure 5. With an interval of 0.1 and ARMA(2,2) dynamic model, the number of whole possible Kalman filters within the parameters space is about 104,976. This is not computationally efficient for a Monte Carlo Simulation, since we run each case 1000 times, we have to limit ourselves to a smaller parameters space. We focus on parameters around the optimal parameters of single Kalman Filter ARMA(2,2) forecasting. Since we run the model for a real time series application only once, we can use all 104,967 filters.

We examine each result based on its data generation model, and later make an overall evaluation. We first compare the averages of statistics of MLE and MMAE forecasting techniques, and the average of percentages of replications MMAE estimates better than MLE estimates, and then we also give the comparison of MMAE with the other techniques later.

We would like to test MMAE's capability to forecast the time series data with big and small variance. We run the same parameterized model twice, one is with big variance and the other is with small variance, as described in Section 4.3. Here we give the summary of those results.

Table 4 shows that MMAE gives 4% better estimates for small variance than big variance for ARMA(2,2) generated data, both compared with MLE. These percentages

alone are not very meaningful. We need to interpret them along with the other statistics averages as well.

Table 4. Percentages of MMAE Estimates Better Than MLE Estimates with ARMA(2,2) Data

	MAE	SSE
Big Variance	40.5%	39.9%
Small Variance	44.4%	43.5%
Average	42.5%	41.7%

From Table 5, ME's are the same, MAE's and SSE's are very close to each other. We also see that, the first step forecast MMAE gives narrower prediction interval with higher coverage. The other prediction intervals of MMAE are also narrower with slightly less coverages than MLE.

Table 5. Averages of Statistics for ARMA(2,2) Data with Big Variance

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	-0.01	1.25	23.48	2.23	0.89	3.03	0.93	2.70	0.91
MMAE	-0.01	1.20	13.86	1.85	0.90	2.48	0.88	2.67	0.89

Although the percentages for the small variance are larger than big variance, the statistics of MMAE for small variance ARMA(2,2) data are slightly worse than MLE's as shown in Table 6. If we evaluate Table 5 and Table 6 together, compared with MLE's statistics, we can conclude that MMAE forecasts for big variance data are better than small variance data. So we have to be careful while interpreting the results. The

percentages do not necessarily mean better forecast and do not reflect the magnitude of the errors exactly. The counter counts the percentages without looking at the quantity of the error; it even counts for very small differences at the bigger decimal places. It is seen that some of the statistics of small variate data are relatively larger than those of big variate data. From this discussion we can conclude that MMAE forecasts are better for more variable data than less variable data.

Table 6. Averages of Statistics for ARMA(2,2) Data with Small Variance

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	-0.11	5.67	262.43	9.88	0.88	12.62	0.92	12.25	0.90
MMAE	-0.14	5.81	291.91	9.83	0.87	10.57	0.86	10.89	0.84

Table 7 shows the averages of Table 5 and Table 6. For the overall average of all ARMA(2,2) data statistics, MMAE's are slightly worse than than MLE's. However prediction interval width for the first step forecast is narrower with the same coverage. The other prediction intervals are also narrower, but have 4% less coverage. As a result, with averages of 42.5% and 41.7% from Table 4, we can conclude that MMAE forecast for an ARMA(2,2) data are about equal to MLE forecasts.

Table 7. Averages of Statistics for all ARMA(2,2) Data

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	-0.06	3.46	142.96	6.06	0.89	7.82	0.92	7.47	0.90
MMAE	-0.08	3.51	152.88	5.84	0.89	6.53	0.87	6.78	0.86

In Table 8, MAE's and ME's are about equal, SSE of MMAE is worse. The first step prediction interval of MMAE is slightly larger with higher coverage, and the other prediction intervals of MMAE are narrower, but have lower coverages. The percentages are very close to 50%. We can say that MMAE forecasts are not better, but equal to MLE's forecasts for AR(1) data.

Table 8. Averages of Statistics and Percentages for AR(1) Data

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	-0.10	2.94	99.70	5.05	0.87	6.50	0.90	6.61	0.90
MMAE	-0.11	3.12	118.48	5.12	0.90	6.08	0.88	6.33	0.87
Percentage		48.3%	47.4						

Table 9 shows that MMAE forecast for AR(2) data are not good compared to MLE for ARMA(2,2). All statistics except first step prediction interval width and coverage are worse than MLE's statistics.

Table 9. Averages of Statistics and Percentages for AR(2) Data

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	-0.10	3.36	119.20	5.75	0.86	7.89	0.92	7.68	0.90
MMAE	-0.11	3.98	273.00	6.48	0.87	6.92	0.87	7.09	0.85
Percentage		41.1%	41.0%						

Although the percentages are less than 50%, Table 10, MMAE forecasts are about equal to MLE forecasts. Only MMAE's MAE and SSE are slightly worse than MLE's.

All the other better statistics and narrower prediction intervals and higher coverages show that MMAE is very good in this case.

Table 10. Averages of Statistics and Percentages for MA(1) Data

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	-0.04	2.59	82.29	4.65	0.86	5.45	0.91	5.46	0.91
MMAE	-0.03	2.63	84.38	5.00	0.87	5.40	0.91	5.50	0.92
Percentage		48.4%	47.9%						

In Table 11, MMAE forecasts are not better but not worse either. The first three statistics seem to be larger, but the prediction intervals are better with almost the same coverages.

Table 11. Averages of Statistics and Percentages for MA(2) Data

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	-0.05	3.37	124.24	5.70	0.91	7.28	0.89	7.25	0.92
MMAE	-0.11	3.61	152.91	6.24	0.91	6.87	0.87	7.02	0.89
Percentage		44.6%	45.5%						

For ARMA (3,3) data, Table 12, MMAE's performance is not as good as MLE's. MMAE's ME, MAE and SSE appear very close to MLE's, the prediction intervals are narrower, but with worse coverages.

Table 12. Averages of Statistics and Percentages for ARMA(3,3) Data

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	0.02	4.30	241.56	5.89	0.85	9.21	0.88	9.66	0.89
MMAE	0.03	4.74	277.67	6.68	0.84	7.64	0.79	7.95	0.86
Percentage		42.1%	41.9%						

Tables 4-12 show the averages of simulation results based on each data generation model. Since the pattern of data of any time series can be any of the above models and MMAE does not require the data to have any specific pattern, we think that if we take the averages of the above models, we can reach a more general conclusion about the goodness of MMAE's forecasting capabilities.

Table 13 shows the averages of all 76 different data generation models; all of them have different variance and sample sizes. The ME and MAE of both methods are almost the same, SSE of MMAE is slightly worse than MLE's. The first step prediction interval of MMAE is only 2%wider than MLE's but with the same coverage probability. The PIW3MSE of MMAE is 14% narrower than MLE's with only 4% worse coverage. The PIW5MSE of MMAE is 8% narrower than MLE's with only 3% worse coverage probability. The narrower prediction intervals automatically affects the prediction interval coverages, make them smaller. Overall averages show that MMAE forecasts are not better than MLE forecasts, but they are not worse either, they are almost equal to each other.

Table 13. Overall Averages of Statistics and Percentages for MMAE and MLE

	ME	MAE	SSE	PIW1	PIC1	PIW3	PIC3	PIW5	PIC5
MLE	-0.05	3.07	118.89	5.21	0.88	6.87	0.91	6.72	0.90
MMAE	-0.07	3.24	149.41	5.33	0.88	6.00	0.87	6.22	0.87
Percentage		44.0%	44.0%						

So far we compare MMAE results with MLE results. Now we compare MMAE with all the other techniques shortly. Tables 14 shows the overall averages of all techniques and Table 15 shows the percentages of MMAE statistics equal or better than other techniques' statistics for 76 runs. Although there is not a best technique in terms of all statistics, AR(2) ULS give more better results than others. When we look at Table 14, all statistics of MMAE are equal or better than other techniques statistics. In terms of ME MMAE is the 8^{th}, in terms of MAE MMAE is the 4^{th}, and in terms of SSE MMAE is the 11^{th} best forecasting technique. The averages of MMAE do not seem better at Table 14, but if we look at the Table 15, MMAE has higher percentages. When we evaluate both table stogether, it is seen that MMAE is much better than all of the classical techniques.

As we highlight the best statistics in each column, MMAE give the best, narrowest 3-step and 5-step prediction interval widths with only 3% less coverage than average coverage. It gives the 3^{rd} best 1-step prediction interval with the same coverage with the others.

As a final word, MMAE forecasts are as good as other well-known techniques, sometimes even better. Especially MMAE's 1-step ahead forecast is better than all of the others. Generally MMAE gives narrowest prediction interval coverages for all forecasts. However as a usual drawback of these narrow intervals its coverages are approximately 3% less than the other techniques coverages, with a theoretical coverage probability of 90%.

Table 14. Overall Averages of Statistics for all Forecasting Techniques

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.07	3.00	117.25	5.03	0.88	6.73	0.92	6.67	0.90
ARMA(2,2)InitialEstimates	-0.06	4.09	192.99	8.30	0.89	8.63	0.89	7.24	0.89
Naive	-0.04	4.21	223.40	8.02	0.89	8.94	0.89	9.03	0.90
Moving Average (T=5)	-0.12	3.45	147.99	6.90	0.88	7.17	0.90	7.31	0.90
Moving Average (T=10)	-0.09	3.40	145.32	6.77	0.88	7.00	0.90	7.12	0.91
Simple Average	-0.07	3.34	140.31	6.79	0.89	6.82	0.90	6.86	0.90
Exponential Smoothing	-0.07	3.72	173.42	6.76	0.87	7.38	0.87	7.48	0.88
Regression	-0.05	3.36	143.75	6.67	0.88	6.67	0.90	6.68	0.90
AR(1)ULS	-0.07	3.26	133.48	6.08	0.88	6.81	0.90	6.68	0.89
AR(2)ULS	-0.06	3.01	114.45	4.98	0.87	7.65	0.94	7.52	0.93
MA(1)ULS	-0.07	3.27	134.63	6.10	0.88	6.70	0.90	6.73	0.90
MA(2)ULS	-0.04	9.02	11196.47	25.72	0.81	6.70	0.90	6.72	0.90
ARMA(1,1)ULS	-0.10	4.02	142.33	7.45	0.88	7.18	0.90	6.99	0.90
ARMA(2,2) ULS	-0.12	4.01	305.74	6.87	0.87	8.57	0.91	7.56	0.90
ARMA(2,2) MLE	-0.05	3.07	118.89	5.21	0.88	6.87	0.91	6.72	0.90
Kalman Filter ARMA(2,2)	-0.06	3.23	145.83	5.50	0.88	6.76	0.90	7.45	0.90
MMAE ARMA(2,2)	-0.07	3.24	149.41	5.33	0.88	6.00	0.87	6.22	0.87

Table 15. Percentages MMAE Is Better Than Other Techniques

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	48.7%	7.9%	9.2%	18.4%	52.6%	65.8%	27.6%	51.3%	42.1%
ARMA(2,2)InitialEstimates	59.2%	82.9%	76.3%	92.1%	52.6%	88.2%	44.7%	57.9%	48.7%
Naive	56.6%	97.4%	93.4%	96.1%	53.9%	97.4%	38.2%	98.7%	48.7%
Moving Average (T=5)	80.3%	82.9%	77.6%	96.1%	60.5%	81.6%	28.9%	73.7%	39.5%
Moving Average (T=10)	75.0%	77.6%	76.3%	97.4%	64.5%	73.7%	32.9%	68.4%	38.2%
Simple Average	48.7%	68.4%	63.2%	97.4%	53.9%	69.7%	34.2%	55.3%	47.4%
Exponential Smoothing	60.5%	93.4%	85.5%	93.4%	69.7%	86.8%	55.3%	85.5%	57.9%
Regression	39.5%	72.4%	63.2%	94.7%	56.6%	64.5%	36.8%	50.0%	47.4%
AR(1)ULS	44.7%	59.2%	51.3%	80.3%	57.9%	64.5%	36.8%	53.9%	48.7%
AR(2)ULS	47.4%	30.3%	27.6%	14.5%	63.2%	77.6%	19.7%	78.9%	21.1%
MA(1)ULS	48.7%	57.9%	51.3%	77.6%	61.8%	65.8%	40.8%	51.3%	50.0%
MA(2)ULS	61.8%	63.2%	60.5%	47.4%	84.2%	65.8%	40.8%	51.3%	50.0%
ARMA(1,1)ULS	50.0%	69.7%	63.2%	89.5%	60.5%	73.7%	34.2%	57.9%	46.1%
ARMA(2,2) ULS	55.3%	42.1%	43.4%	35.5%	76.3%	71.1%	28.9%	56.6%	48.7%
ARMA(2,2) MLE	46.1%	28.9%	27.6%	26.3%	67.1%	73.7%	26.3%	59.2%	43.4%
Kalman Filter ARMA(2,2)	56.6%	43.4%	43.4%	44.7%	61.8%	67.1%	34.2%	63.2%	48.7%

CHAPTER V: CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

A new contribution has been made to time series forecasting. Our method is more robust and applicable to any kind of time series with less effort compared to most other methods. We are saying robust, because we can fit MMAE to any kind of data, as we fit it to AR(p), MA(q), and complex ARMA (3,3) generated data. Even though we use only small portion of the possible parameter space, the MMAE forecasts are as good as any other well-known methods. The simulation results given at Tables in Appendix B and the analysis in Chapter IV show that most of the time MMAE statistics, ME, MAE, SSE, are much better than most of the other methods'. Moreover, in some cases while some of the methods give poor forecasts when the data generation parameters are redundant, MMAE has never this problem (see Appendix B Table 98). MMAE can adapt for changing parameters and variances.

As a formal comparison, we compare MMAE forecasts with ARMA (2,2) MLE (maximum likelihood estimation) forecasts and give the results in Section 4.5. The average of percentages of the results presented in Section 4.5 and in the Appendix B is about 45%, which is very close to 50%. We can conclude that MMAE forecasts are as good as MLE forecasts, which is considered to be the standard and hard to beat.

The best part of MMAE forecast is: at the confidence level of α=0.1, almost every time it gives the narrowest prediction intervals. Although MMAE's prediction intervals are narrower than the others', its prediction interval coverages are just like the

others', around theoretical coverage of 90%; the narrower the prediction interval, the less the uncertainty in forecasting.

All these simulation results and statistics show that the MMAE algorithm, which is taken from engineers, is worth of using in time series forecasting. This is especially true when we have to make forecasts frequently on a daily base or weekly or monthly base, the automated algorithm of MMAE comes out superior to others. To apply other methods, we have to calculate autocorrelation and partial autocorrelation functions of time series data and identify which model is more appropriate. Later, once the model is identified we have to search for the optimal parameters of the model, then we can forecast. Each time we get a new datum we just cannot say that the old model is still applicable. We have to go through whole process from beginning to end. As we talk about the advantages of MMAE in Chapter II and Chapter III, we say that we do not need the model identification and parameter estimation steps. MMAE algorithm automates those steps with Bayesian probabilistic weighting. We simply insert the data, run the model and get the results. Whenever we get a new datum, Kalman filter algorithm in MMAE adjusts to it and updates the estimate.

Another benefit of MMAE is that, while the other methods use some sort of nonlinear search and estimation methods (like least square estimation or maximum likelihood estimation) to estimate their unknown parameters [7], MMAE does not need any kind of parameter estimation or search techniques. It simply uses all possible parameters in the parameter space. With an interval of 0.1 for parameters, MMAE uses more than 100,000 models, weights them probabilistically, uses the most likely models and gives a forecast, which is probabilistically weighted mean of all models' forecasts.

As a final verification, we applied MMAE in time series forecasting and obtained good results in a Monte Carlo simulation. We recommend MMAE when forecasting is done on regular basis. Its automatic capability allows people to easily make forecast as good as most other complex methods, without knowing anything about time series forecasting methods.

5.2 Recommendations

The MMAE algorithm developed in this research is a viable method for time series analysis. There are some points that warrant further research.

1. We use a fixed data mean while generating data. This allows us to forecast only stationary time series. However, most real time series are non-stationary and seasonal. We could build a model to estimate the data mean from data along with the time, for non-stationary time series. We could also build seasonal Kalman filter to include in MMAE bank.

2. We calculate prediction intervals according to Makridas and others [15]. They can also be calculated according to Box-Jenkins [3:142-145].

3. Each filter in MMAE bank is started in a conditional manner, providing starting prior values and shocks. Another approach could be an unconditional manner, which requires backward forecasting with each filter to determine the prior observations before beginning forward estimation.

APPENDIX A: Data Generation Parameters

Table 16. ARMA (2,2) Data Generation Parameters

						Noise Std. Dev	
Case	Phi 1	Phi 2	Theta 1	Theta 2	Sample Size	Dynamic	Measurement
1	0.4	0.4	0.2	0.2	300	1	0.1
2	0.4	0.3	-0.4	0.3	300	1	0.1
3	0.5	0.3	-0.4	-0.5	300	1	0.1
4	0.4	0.5	-0.7	-0.5	200	1	0.1
5	0.7	0.2	0.8	-0.1	300	1	0.1
6	0.3	0.2	0.6	-0.4	200	1	0.1
7	-0.4	0.2	0.5	0.3	200	1	0.1
8	-0.4	0.2	0.5	0.3	400	1	0.1
9	-0.2	0.1	-0.4	0.5	300	1	0.1
10	-0.1	0.1	-0.6	-0.4	800	1	0.1
11	-0.2	0.6	-0.5	-0.5	200	1	0.1
12	-0.3	0.3	0.8	-0.1	300	1	0.1
13	-0.7	-0.6	0.2	0.2	200	1	0.1
14	-0.7	-0.6	-0.4	0.5	300	1	0.1
15	-0.7	-0.7	-0.3	-0.3	100	1	0.1
16	1	-0.6	0.4	0.2	200	1	0.1
17	0.6	-0.5	-0.3	0.3	500	1	0.1
18	0.6	-0.5	-0.3	0.3	300	1	0.1
19	0.6	-0.5	-0.6	-0.5	600	1	0.1
20	0.6	-0.5	-0.6	-0.5	400	1	0.1
21	0.6	-0.5	-0.6	-0.5	200	1	0.1
22	0.3	-0.3	1.4	-0.7	200	1	0.1
23	0.4	-0.3	1	-0.6	300	1	0.1
24	0.4	0.4	0.2	0.2	300	5	1
25	0.4	0.3	-0.4	0.3	300	5	1
26	0.5	0.3	-0.4	-0.5	300	5	1
27	0.7	0.2	0.8	-0.1	300	5	1
28	-0.4	0.2	0.5	0.3	400	5	1
29	-0.2	0.1	-0.4	0.5	300	5	1
30	-0.2	0.6	-0.5	-0.5	200	5	1
31	-0.7	-0.6	-0.4	0.5	300	5	1
32	-0.7	-0.7	-0.3	-0.3	100	5	1
33	1	-0.6	0.4	0.2	200	5	1
34	0.6	-0.5	-0.3	0.3	300	5	1
35	0.6	-0.5	-0.6	-0.5	400	5	1
36	0.4	-0.3	1	-0.6	300	5	1

Table 17. AR (1) Data Generation Parameters

Case	Phi 1	Phi 2	Theta 1	Theta 2	Sample Size	Noise Std. Dev. Dynamic	Noise Std. Dev. Measurement
37	0.5	0	0	0	300	5	1.0
38	-0.5	0	0	0	200	1	0.1
39	0.5	0	0	0	200	1	0.1
40	-0.5	0	0	0	300	5	1.0
41	0.8	0	0	0	300	5	1.0
42	0.8	0	0	0	300	1	0.1
43	0.8	0	0	0	300	5	1.0
44	0.8	0	0	0	300	1	0.1
45	-0.8	0	0	0	300	1	0.1
46	-0.2	0	0	0	300	5	1.0

Table 18. AR (2) Data Generation Parameters

Case	Phi 1	Phi 2	Theta 1	Theta 2	Sample Size	Noise Std. Dev. Dynamic	Noise Std. Dev. Measurement
47	0.9	-0.6	0	0	300	5	1.0
48	-0.9	-0.6	0	0	300	5	1.0
49	0.5	0.2	0	0	300	5	1.0
50	-0.5	0.2	0	0	300	5	1.0
51	-0.5	-0.5	0	0	200	5	1.0
52	0.5	-0.5	0	0	200	5	1.0
53	0.5	0.2	0	0	300	1	0.1
54	-0.5	0.2	0	0	300	1	0.1
55	-0.5	-0.5	0	0	200	1	0.1
56	0.5	-0.5	0	0	200	1	0.1

Table 19. MA (1) Data Generation Parameters

Case	Phi 1	Phi 2	Theta 1	Theta 2	Sample Size	Noise Std. Dev. Dynamic	Noise Std. Dev. Measurement
57	0	0	-0.9	0	500	5	1
58	0	0	-0.9	0	500	1	0
59	0	0	0.9	0	500	1	0
60	0	0	-0.5	0	200	5	1
61	0	0	-0.5	0	200	5	1
62	0	0	0.5	0	200	1	0
63	0	0	-0.5	0	200	1	0

Table 20. MA (2) Data Generation Parameters

Case	Phi 1	Phi 2	Theta 1	Theta 2	Sample Size	Noises	
						Dynamic	Measurement
64	0	0	0.5	0.4	400	5	1.0
65	0	0	-0.5	0.3	400	5	1.0
66	0	0	-1.3	-0.7	300	5	1.0
67	0	0	0.5	-0.3	400	5	1.0
68	0	0	0.5	0.4	400	1	0.1
69	0	0	-0.5	0.3	400	1	0.1
70	0	0	-1.3	-0.7	400	1	0.1

Table 21. ARMA (3,3) Data Generation Parameters

Case	Phi 1	Phi 2	Phi3	Theta 1	Theta 2	Theta3	Sample Size	Noise Std. Dev.	
								Dynamic	Measure_ment
71	0.9	-0.6	0.5	-0.8	-0.5	0.2	200	5	1.0
72	0.9	-0.6	0.5	-0.8	-0.5	0.2	200	1	0.1
73	0.4	0.3	0.1	0.7	-0.3	0.4	200	5	1.0
74	0.4	0.3	0.1	0.7	-0.3	0.4	200	1	0.1
75	0.7	-0.8	0.3	0.6	0.3	-0.8	200	1	0.1
76	0.7	-0.8	0.3	0.6	0.3	-0.8	200	5	1.0

APPENDIX B: Monte Carlo Simulation Results

Table 22. Monte Carlo Simulation Results for Case 1

TRUE PARAMETERS

Phi1 =	0.4	Sample Size =	300
Phi2 =	0.4	Number of Predictions=	5
Theta1=	0.2	Noise Std. Deviation =	1
Theta2=	0.2	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.04	0.85	5.51	1.73	0.92	1.79	0.91	1.81	0.91
ARMA(2,2)InitialEstimates	-0.12	1.73	25.78	4.73	0.93	3.01	0.91	2.14	0.84
Naive	-0.02	1.09	9.86	2.14	0.87	2.23	0.89	2.32	0.88
Moving Average (T=5)	-0.07	0.90	6.22	1.74	0.85	1.92	0.92	2.01	0.90
Moving Average (T=10)	-0.06	0.92	6.60	1.77	0.83	1.91	0.92	1.97	0.89
Simple Average	-0.03	0.93	6.63	1.86	0.88	1.88	0.92	1.90	0.87
Exponential Smoothing	-0.04	0.96	7.17	1.73	0.85	1.87	0.88	1.97	0.87
Regression	-0.02	0.94	6.88	1.82	0.88	1.82	0.91	1.82	0.87
AR(1)ULS	-0.03	0.91	6.42	1.75	0.88	1.78	0.89	1.80	0.86
AR(2)ULS	-0.04	0.86	5.75	1.66	0.91	1.76	0.91	1.82	0.88
MA(1)ULS	-0.03	0.92	6.55	1.78	0.87	1.84	0.91	1.84	0.86
MA(2)ULS	-0.03	0.91	6.36	1.71	0.93	1.84	0.91	1.84	0.86
ARMA(1,1)ULS	-0.03	0.93	6.56	1.79	0.88	1.93	0.92	2.01	0.91
ARMA(2,2) ULS	-0.03	0.90	6.21	1.71	0.91	1.84	0.91	1.84	0.85
ARMA(2,2) MLE	-0.03	0.85	5.56	1.76	0.91	1.80	0.92	1.82	0.88
Kalman Filter ARMA(2,2)	-0.05	0.87	5.79	1.67	0.89	1.77	0.91	1.80	0.87
MMAE ARMA(2,2)	-0.03	0.86	5.63	1.76	0.91	1.88	0.94	1.95	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	49.3	48.4

Table 23. Monte Carlo Simulation Results for Case 2

TRUE PARAMETERS

Phi1 =	0.4		Sample Size =	300
Phi2 =	0.3		Number of Predictions=	5
Theta1=	-0.4		Noise Std. Deviation =	1
Theta2=	0.3		Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.07	1.04	8.37	1.79	0.93	2.24	0.90	2.28	0.91
ARMA(2,2)InitialEstimates	-0.05	1.27	12.58	2.10	0.89	2.54	0.88	2.35	0.89
Naive	-0.05	1.26	12.58	1.89	0.90	2.64	0.89	2.92	0.90
Moving Average (T=5)	-0.11	1.21	11.17	2.12	0.85	2.59	0.92	2.72	0.92
Moving Average (T=10)	-0.10	1.23	11.79	2.23	0.87	2.53	0.89	2.62	0.93
Simple Average	-0.06	1.18	10.75	2.33	0.87	2.38	0.92	2.40	0.89
Exponential Smoothing	-0.07	1.20	11.39	1.87	0.91	2.47	0.91	2.72	0.87
Regression	-0.03	1.21	11.24	2.28	0.87	2.28	0.90	2.28	0.86
AR(1)ULS	-0.06	1.13	9.95	1.99	0.89	2.18	0.85	2.26	0.88
A[illegible] 53 [illegible]S	-0.06	1.05	8.62	1.71	0.92	2.29	0.88	2.50	0.92
M[illegible]S	-0.06	1.15	10.15	2.02	0.88	2.31	0.89	2.31	0.88
MA(2)ULS	-0.06	1.11	9.69	1.74	0.89	2.31	0.89	2.31	0.88
ARMA(1,1)ULS	-0.07	1.12	9.58	1.88	0.89	2.46	0.91	2.56	0.92
ARMA(2,2) ULS	-0.06	1.10	9.46	1.73	0.90	2.31	0.88	2.31	0.89
ARMA(2,2) MLE	-0.08	1.03	8.34	1.75	0.90	2.24	0.90	2.28	0.91
Kalman Filter ARMA(2,2)	-0.07	1.04	8.42	1.68	0.89	2.17	0.87	2.28	0.90
MMAE ARMA(2,2)	-0.07	1.05	8.58	1.79	0.92	2.29	0.91	2.40	0.92

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	42.7	40.4

Table 24. Monte Carlo Simulation Results for Case 3

TRUE PARAMETERS

Phi1 =	0.5		Sample Size =	300
Phi2 =	0.3		Number of Predictions=	5
Theta1=	-0.4		Noise Std. Deviation =	1
Theta2=	-0.5		Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.06	1.40	16.39	1.82	0.93	3.38	0.90	3.73	0.88
ARMA(2,2)InitialEstimates	-0.07	1.83	26.83	3.12	0.87	3.68	0.91	3.87	0.87
Naive	-0.11	1.65	21.70	1.91	0.82	3.34	0.86	4.16	0.90
Moving Average (T=5)	-0.14	1.94	29.40	2.78	0.84	4.07	0.91	4.52	0.91
Moving Average (T=10)	-0.10	2.08	34.54	3.37	0.86	4.22	0.90	4.54	0.91
Simple Average	-0.05	2.20	35.93	4.08	0.91	4.21	0.89	4.28	0.90
Exponential Smoothing	-0.12	1.66	21.75	2.04	0.83	3.39	0.89	4.14	0.91
Regression	0.01	2.26	39.25	3.94	0.89	3.94	0.87	3.94	0.88
AR(1)ULS	-0.05	2.08	32.71	3.22	0.90	3.49	0.80	3.69	0.84
AR(53	-0.10	1.52	18.58	1.83	0.80	3.19	0.86	3.95	0.89
MA	-0.05	2.13	33.87	3.37	0.90	4.04	0.89	4.04	0.89
MA(2)ULS	-0.04	1.98	30.87	2.11	0.89	4.04	0.89	4.04	0.89
ARMA(1,1)ULS	-0.06	1.66	21.58	2.07	0.85	3.66	0.91	4.14	0.91
ARMA(2,2) ULS	-0.04	1.84	27.08	2.06	0.88	3.99	0.89	4.04	0.89
ARMA(2,2) MLE	-0.03	1.40	16.65	1.79	0.88	3.36	0.92	3.74	0.90
Kalman Filter ARMA(2,2)	-0.03	1.41	16.98	1.71	0.88	3.07	0.88	3.62	0.89
MMAE ARMA(2,2)	-0.05	1.47	17.69	1.82	0.89	3.11	0.86	3.68	0.87

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	35.1	37.5

Table 25. Monte Carlo Simulation Results for Case 4

TRUE PARAMETERS

Phi1 =	0.4	Sample Size =	200
Phi2 =	0.5	Number of Predictions=	5
Theta1=	-0.7	Noise Std. Deviation =	1
Theta2=	-0.5	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.01	1.56	20.04	3.15	1.00	4.63	0.98	5.04	0.98
ARMA(2,2)InitialEstimates	-0.20	2.69	59.89	4.01	0.90	5.65	0.88	5.74	0.89
Naive	0.03	1.66	22.62	1.88	0.87	3.59	0.86	4.61	0.90
Moving Average (T=5)	0.08	2.04	33.49	3.04	0.87	4.65	0.91	5.34	0.91
Moving Average (T=10)	0.16	2.24	42.13	3.91	0.90	5.08	0.92	5.64	0.91
Simple Average	-0.08	3.34	86.02	5.88	0.82	6.16	0.85	6.31	0.85
Exponential Smoothing	0.04	1.70	23.56	2.13	0.88	3.72	0.88	4.68	0.90
Regression	-0.37	3.39	89.08	5.41	0.82	5.41	0.82	5.41	0.82
AR(1)ULS	-0.07	3.16	77.51	4.54	0.81	4.78	0.75	4.98	0.75
AR(2)ULS	0.04	1.66	22.27	1.84	0.87	3.49	0.87	4.47	0.94
MA(1)ULS	-0.07	3.22	80.62	4.78	0.81	5.81	0.82	5.80	0.81
MA(2)ULS	-0.08	2.96	72.35	2.61	0.87	5.81	0.82	5.80	0.81
ARMA(1,1)ULS	-0.01	1.72	24.43	1.98	0.89	4.21	0.91	5.14	0.91
ARMA(2,2) ULS	-0.16	2.97	73.41	3.03	0.84	5.93	0.81	5.92	0.81
ARMA(2,2) MLE	0.07	1.73	24.72	2.84	0.94	4.42	0.95	4.89	0.94
Kalman Filter ARMA(2,2)	0.12	1.75	25.56	1.82	0.87	3.66	0.83	4.41	0.90
MMAE ARMA(2,2)	0.04	1.72	24.34	1.84	0.84	3.43	0.84	4.25	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	55.4	57.1

Table 26. Monte Carlo Simulation Results for Case 5

TRUE PARAMETERS

Phi1 =	0.7	Sample Size =	300
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0.8	Noise Std. Deviation =	1
Theta2=	-0.1	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.03	0.81	5.14	2.15	0.96	1.85	0.93	1.84	0.91
ARMA(2,2)InitialEstimates	-0.08	1.91	34.43	8.51	0.95	2.47	0.85	1.90	0.84
Naive	0.00	1.14	10.87	2.54	0.87	2.28	0.91	2.30	0.90
Moving Average (T=5)	-0.04	0.84	5.54	1.77	0.84	1.79	0.93	1.84	0.91
Moving Average (T=10)	-0.04	0.85	5.71	1.74	0.86	1.78	0.92	1.82	0.89
Simple Average	-0.03	0.89	6.16	1.82	0.90	1.82	0.92	1.83	0.90
Exponential Smoothing	-0.02	0.94	7.33	1.88	0.84	1.79	0.86	1.84	0.88
Regression	-0.01	0.90	6.33	1.78	0.89	1.78	0.90	1.78	0.88
AR(1)ULS	-0.02	0.89	6.15	1.81	0.88	1.82	0.91	1.83	0.87
AR(2)ULS	-0.02	0.86	5.75	1.71	0.90	1.76	0.91	1.79	0.88
MA(1)ULS	-0.03	0.90	6.27	1.84	0.88	1.80	0.92	1.80	0.88
MA(2)ULS	-0.02	0.88	6.00	1.73	0.91	1.80	0.92	1.80	0.88
ARMA(1,1)ULS	-5.99	8.04	11.62	14.92	0.83	6.09	0.85	3.23	0.87
ARMA(2,2) ULS	-0.02	0.87	5.92	1.88	0.91	1.82	0.91	1.80	0.88
ARMA(2,2) MLE	-0.03	0.81	5.18	2.39	0.94	2.00	0.94	1.96	0.91
Kalman Filter ARMA(2,2)	-0.03	0.86	6.77	1.67	0.90	1.83	0.92	2.27	0.89
MMAE ARMA(2,2)	-0.03	0.86	6.66	1.85	0.89	1.94	0.93	1.98	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	43.8	47

Table 27. Monte Carlo Simulation Results for Case 6

TRUE PARAMETERS

Phi1 =	0.3	Sample Size =	200
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0.6	Noise Std. Deviation =	1
Theta2=	-0.4	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.90	6.32	1.68	0.83	1.94	0.91	1.94	0.93
ARMA(2,2)InitialEstimates	0.08	2.77	25.58	9.41	0.87	3.38	0.80	2.40	0.88
Naive	-0.01	1.24	12.26	3.12	0.90	2.67	0.89	2.68	0.90
Moving Average (T=5)	-0.02	0.98	7.63	2.03	0.89	2.08	0.93	2.12	0.91
Moving Average (T=10)	0.04	0.98	7.51	2.01	0.88	2.04	0.92	2.06	0.94
Simple Average	0.00	0.96	7.16	1.97	0.89	1.96	0.90	1.97	0.95
Exponential Smoothing	-0.01	1.03	8.66	2.18	0.84	2.08	0.89	2.13	0.87
Regression	0.00	0.96	7.19	1.93	0.87	1.93	0.89	1.93	0.92
AR(1)ULS	0.00	0.95	7.04	1.85	0.86	2.01	0.92	2.00	0.95
AR(2)ULS	0.00	0.93	6.72	1.72	0.84	2.07	0.92	2.12	0.95
MA(1)ULS	0.00	0.96	7.17	1.89	0.87	1.93	0.90	1.93	0.93
MA(2)ULS	0.00	0.95	6.98	1.67	0.83	1.93	0.90	1.93	0.93
ARMA(1,1)ULS	-0.13	1.16	11.13	2.67	0.83	2.10	0.92	1.94	0.95
ARMA(2,2) ULS	0.14	2.19	23.75	5.78	0.83	3.27	0.90	2.39	0.91
ARMA(2,2) MLE	0.14	2.17	23.74	5.78	0.82	3.27	0.91	2.39	0.91
Kalman Filter ARMA(2,2)	0.01	0.92	6.57	1.67	0.80	1.95	0.93	1.96	0.93
MMAE ARMA(2,2)	0.01	0.92	6.68	1.83	0.87	2.18	0.95	2.24	0.96

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	58	54

Table 28. Monte Carlo Simulation Results for Case 7

TRUE PARAMETERS

Phi1 =	-0.4	Sample Size =	200
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0.5	Noise Std. Deviation =	1
Theta2=	0.3	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.08	9.26	1.70	0.87	2.39	0.97	2.36	0.90
ARMA(2,2)InitialEstimates	0.00	1.27	12.30	2.24	0.87	2.56	0.91	2.42	0.91
Naive	-0.01	1.71	23.01	4.26	0.90	3.72	0.92	3.49	0.88
Moving Average (T=5)	-0.02	1.20	11.28	2.63	0.89	2.36	0.88	2.39	0.87
Moving Average (T=10)	0.00	1.18	10.84	2.48	0.91	2.35	0.91	2.36	0.89
Simple Average	0.00	1.16	10.57	2.40	0.90	2.35	0.92	2.35	0.90
Exponential Smoothing	-0.01	1.39	15.34	2.91	0.82	2.63	0.85	2.54	0.82
Regression	0.00	1.16	10.60	2.36	0.90	2.36	0.93	2.37	0.89
AR(1)ULS	0.00	1.13	10.05	2.04	0.88	2.28	0.91	2.36	0.90
AR(2)ULS	0.00	1.11	9.77	1.77	0.85	2.47	0.95	2.67	0.94
MA(1)ULS	0.00	1.14	10.15	2.07	0.88	2.35	0.92	2.35	0.89
MA(2)ULS	0.00	1.12	9.86	1.71	0.84	2.35	0.92	2.35	0.89
ARMA(1,1)ULS	-0.01	1.14	10.21	2.22	0.88	2.38	0.94	2.37	0.89
ARMA(2,2) ULS	0.00	1.10	9.49	1.69	0.84	2.38	0.95	2.36	0.90
ARMA(2,2) MLE	0.00	1.12	9.85	1.73	0.85	2.42	0.95	2.38	0.90
Kalman Filter ARMA(2,2)	-0.01	1.16	10.50	1.72	0.86	2.42	0.92	2.53	0.93
MMAE ARMA(2,2)	-0.02	1.17	10.83	1.79	0.87	2.40	0.91	2.46	0.89

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	53.4	53.5

Table 29. Monte Carlo Simulation Results for Case 8

TRUE PARAMETERS

Phi1 =	-0.4	Sample Size =	400
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0.5	Noise Std. Deviation =	1
Theta2=	0.3	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.01	1.06	8.85	1.68	0.91	2.38	0.93	2.35	0.88
ARMA(2,2)InitialEstimates	0.02	1.33	13.63	2.30	0.89	2.53	0.82	2.40	0.90
Naive	0.07	1.83	26.56	4.26	0.87	3.72	0.87	3.49	0.86
Moving Average (T=5)	0.01	1.27	12.28	2.63	0.87	2.35	0.85	2.38	0.90
Moving Average (T=10)	0.02	1.24	11.73	2.47	0.87	2.34	0.85	2.35	0.91
Simple Average	0.01	1.21	11.36	2.37	0.87	2.35	0.91	2.35	0.90
Exponential Smoothing	0.04	1.48	17.15	2.90	0.81	2.63	0.83	2.53	0.83
Regression	0.01	1.21	11.38	2.36	0.86	2.36	0.91	2.36	0.90
AR(1)ULS	0.01	1.17	10.54	2.04	0.84	2.27	0.87	2.35	0.90
AR(2)ULS	0.01	1.11	9.45	1.77	0.90	2.47	0.92	2.66	0.94
MA(1)ULS	0.01	1.18	10.73	2.06	0.85	2.35	0.91	2.35	0.90
MA(2)ULS	0.01	1.13	10.01	1.71	0.90	2.35	0.91	2.35	0.90
ARMA(1,1)ULS	0.01	1.21	11.20	2.27	0.86	2.36	0.91	2.35	0.90
ARMA(2,2) ULS	0.01	1.07	8.94	1.68	0.90	2.38	0.92	2.35	0.90
ARMA(2,2) MLE	0.01	1.08	9.09	1.69	0.90	2.41	0.92	2.37	0.91
Kalman Filter ARMA(2,2)	0.01	1.12	9.81	1.70	0.91	2.38	0.89	2.46	0.90
MMAE ARMA(2,2)	0.01	1.13	9.83	1.79	0.91	2.39	0.88	2.47	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	42	39.6

Table 30. Monte Carlo Simulation Results for Case 9

TRUE PARAMETERS

Phi1 =	-0.2	Sample Size =	300
Phi2 =	0.1	Number of Predictions=	5
Theta1=	-0.4	Noise Std. Deviation =	1
Theta2=	0.5	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.03	0.83	5.48	1.70	0.89	1.86	0.93	1.85	0.89
ARMA(2,2)InitialEstimates	-0.02	0.94	7.02	1.96	0.86	1.94	0.92	1.85	0.91
Naive	0.02	1.32	14.10	2.56	0.89	2.54	0.93	2.59	0.88
Moving Average (T=5)	-0.04	0.93	6.87	2.06	0.90	1.99	0.95	1.98	0.92
Moving Average (T=10)	-0.05	0.90	6.35	1.95	0.93	1.91	0.93	1.91	0.91
Simple Average	-0.03	0.87	5.94	1.87	0.93	1.86	0.93	1.86	0.90
Exponential Smoothing	0.00	1.10	9.67	2.13	0.89	2.02	0.88	2.03	0.87
Regression	-0.02	0.87	5.95	1.85	0.93	1.85	0.93	1.85	0.89
AR(1)ULS	-0.03	0.87	6.00	1.88	0.91	1.90	0.93	1.90	0.90
AR(2)ULS	-0.03	0.87	5.92	1.74	0.91	1.92	0.94	1.93	0.92
MA(1)ULS	-0.03	0.87	5.92	1.85	0.92	1.85	0.91	1.85	0.90
MA(2)ULS	-0.03	0.86	5.91	1.68	0.88	1.85	0.91	1.85	0.90
ARMA(1,1)ULS	-0.03	0.87	6.00	1.87	0.93	1.85	0.91	1.85	0.90
ARMA(2,2) ULS	-0.03	0.84	5.55	1.71	0.88	1.85	0.92	1.85	0.90
ARMA(2,2) MLE	-0.04	0.84	5.59	1.79	0.88	1.87	0.91	1.85	0.89
Kalman Filter ARMA(2,2)	-0.04	0.85	5.68	1.67	0.87	1.87	0.92	1.86	0.90
MMAE ARMA(2,2)	-0.03	0.85	5.78	1.76	0.89	1.93	0.94	1.94	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	46.1	44.2

Table 31. Monte Carlo Simulation Results for Case 10

TRUE PARAMETERS

Phi1 =	-0.1	Sample Size =	800
Phi2 =	0.1	Number of Predictions=	5
Theta1=	-0.6	Noise Std. Deviation =	1
Theta2=	-0.4	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.95	7.21	1.65	0.91	1.98	0.87	1.98	0.91
ARMA(2,2)InitialEstimates	-0.02	1.07	9.31	2.63	0.92	2.07	0.89	1.99	0.94
Naive	0.02	1.30	13.14	2.00	0.87	2.78	0.89	2.81	0.90
Moving Average (T=5)	0.03	1.06	9.17	2.08	0.92	2.37	0.88	2.38	0.88
Moving Average (T=10)	0.02	1.03	8.46	2.06	0.93	2.21	0.91	2.22	0.93
Simple Average	-0.01	0.98	7.59	1.99	0.91	2.00	0.87	2.00	0.91
Exponential Smoothing	0.01	1.21	11.55	1.87	0.91	2.47	0.85	2.50	0.89
Regression	-0.01	0.97	7.55	1.98	0.89	1.98	0.87	1.98	0.92
AR(1)ULS	-0.01	0.96	7.36	1.80	0.88	2.03	0.87	2.04	0.91
AR(2)ULS	-0.01	0.95	7.37	1.71	0.90	2.20	0.90	2.22	0.94
MA(1)ULS	-0.01	0.96	7.43	1.83	0.88	1.98	0.86	1.98	0.91
MA(2)ULS	-0.01	0.95	7.29	1.65	0.91	1.98	0.86	1.98	0.91
ARMA(1,1)ULS	-0.01	0.97	7.52	1.94	0.88	1.98	0.87	1.98	0.91
ARMA(2,2) ULS	-0.01	0.95	7.21	1.65	0.91	1.98	0.87	1.98	0.91
ARMA(2,2) MLE	-0.01	1.01	8.44	2.00	0.93	2.12	0.86	2.01	0.90
Kalman Filter ARMA(2,2)	-0.04	1.00	8.23	1.92	0.92	2.09	0.87	2.02	0.89
MMAE ARMA(2,2)	0.00	0.97	7.45	1.87	0.93	1.87	0.86	1.87	0.89

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	53.5	55.5

Table 32. Monte Carlo Simulation Results for Case 11

TRUE PARAMETERS

Phi1 =	-0.2	Sample Size =	200
Phi2 =	0.6	Number of Predictions=	5
Theta1=	-0.5	Noise Std. Deviation =	1
Theta2=	-0.5	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	1.06	8.98	1.77	0.87	2.50	0.88	2.68	0.94
ARMA(2,2)InitialEstimates	0.00	1.30	13.23	2.86	0.95	2.59	0.86	2.72	0.94
Naive	-0.10	1.61	21.45	3.92	0.93	4.25	0.87	4.32	0.90
Moving Average (T=5)	-0.04	1.47	16.66	2.86	0.92	3.01	0.88	3.15	0.92
Moving Average (T=10)	0.07	1.43	15.97	2.82	0.92	3.02	0.93	3.09	0.91
Simple Average	-0.01	1.39	15.37	2.84	0.90	2.87	0.91	2.89	0.92
Exponential Smoothing	-0.06	1.46	16.72	2.81	0.89	3.23	0.85	3.37	0.89
Regression	-0.02	1.38	15.55	2.79	0.91	2.79	0.91	2.79	0.91
AR(1)ULS	-0.02	1.39	15.16	2.78	0.89	2.80	0.90	2.81	0.92
AR(2)ULS	0.00	1.14	10.11	1.86	0.85	2.60	0.88	2.94	0.95
MA(1)ULS	-0.02	1.41	15.65	2.90	0.88	2.80	0.89	2.80	0.91
MA(2)ULS	-0.01	1.34	14.02	1.98	0.88	2.80	0.89	2.80	0.91
ARMA(1,1)ULS	-0.01	1.37	14.88	2.71	0.90	2.84	0.89	2.80	0.91
ARMA(2,2) ULS	-0.01	1.18	10.87	1.82	0.87	2.79	0.90	2.79	0.92
ARMA(2,2) MLE	-0.01	1.14	10.14	1.81	0.86	2.62	0.88	2.73	0.95
Kalman Filter ARMA(2,2)	-0.02	1.25	12.31	1.75	0.85	2.96	0.89	2.83	0.93
MMAE ARMA(2,2)	-0.01	1.24	12.00	1.83	0.86	2.52	0.88	2.71	0.92

53

PERCENT OF ESTIMATES :R THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	29.9	26.7

Table 33. Monte Carlo Simulation Results for Case 12

TRUE PARAMETERS

Phi1 =	-0.3	Sample Size =	300
Phi2 =	0.3	Number of Predictions=	5
Theta1=	0.8	Noise Std. Deviation =	1
Theta2=	-0.1	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	1.19	11.52	1.69	0.87	3.03	0.94	3.02	0.92
ARMA(2,2)InitialEstimates	-0.01	1.49	16.87	2.89	0.88	3.59	0.94	3.63	0.93
Naive	0.05	2.18	42.13	5.76	0.89	5.08	0.86	4.68	0.89
Moving Average (T=5)	0.00	1.53	18.45	3.39	0.90	2.96	0.87	3.03	0.87
Moving Average (T=10)	-0.01	1.50	17.75	3.18	0.88	2.99	0.90	3.02	0.89
Simple Average	0.00	1.48	17.31	3.07	0.90	3.01	0.91	3.02	0.90
Exponential Smoothing	0.03	1.75	26.39	3.77	0.81	3.48	0.81	3.31	0.82
Regression	0.00	1.48	17.34	3.03	0.88	3.03	0.91	3.04	0.90
AR(1)ULS	0.00	1.41	15.53	2.48	0.88	2.81	0.88	2.96	0.88
AR(2)ULS	-0.01	1.19	11.61	1.69	0.89	2.98	0.94	3.47	0.94
MA(1)ULS	0.00	1.43	16.04	2.55	0.87	3.02	0.91	3.02	0.89
MA(2)ULS	-0.01	1.35	14.48	1.80	0.84	3.02	0.91	3.02	0.89
ARMA(1,1)ULS	-0.02	1.42	15.60	2.78	0.88	3.09	0.92	3.06	0.90
ARMA(2,2) ULS	-0.01	1.20	11.62	1.67	0.88	3.13	0.94	3.04	0.92
ARMA(2,2) MLE	0.00	1.21	11.94	1.68	0.87	3.84	0.95	3.75	0.94
Kalman Filter ARMA(2,2)	-0.07	2.24	65.59	1.72	0.89	4.16	0.90	8.27	0.88
MMAE ARMA(2,2)	-0.06	2.36	75.30	1.96	0.93	2.91	0.79	3.13	0.55

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	28.7	27.9

Table 34. Monte Carlo Simulation Results for Case 13

TRUE PARAMETERS

Phi1 =	-0.7	Sample Size =	200
Phi2 =	-0.6	Number of Predictions=	5
Theta1=	0.2	Noise Std. Deviation =	1
Theta2=	0.2	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.13	10.18	1.67	0.83	3.05	0.99	2.66	0.89
ARMA(2,2)InitialEstimates	0.11	7.27	1400.15	11.28	0.83	25.05	1.00	9.99	0.89
Naive	-0.05	1.88	28.39	4.50	0.89	2.62	0.94	4.11	0.92
Moving Average (T=5)	-0.05	1.32	13.85	3.02	0.89	2.84	0.98	2.62	0.87
Moving Average (T=10)	0.00	1.29	13.14	2.79	0.88	2.72	0.95	2.63	0.87
Simple Average	-0.01	1.27	12.72	2.70	0.90	2.67	0.93	2.65	0.87
Exponential Smoothing	-0.04	1.54	18.90	3.28	0.86	2.34	0.95	2.88	0.88
Regression	-0.02	1.27	12.77	2.66	0.87	2.66	0.93	2.66	0.87
AR(1)ULS	-0.01	1.26	12.43	2.44	0.91	3.02	0.96	2.60	0.85
AR(2)ULS	0.00	1.17	10.73	1.70	0.83	3.88	1.00	3.35	0.98
MA(1)ULS	-0.01	1.25	12.22	2.39	0.89	2.65	0.92	2.65	0.87
MA(2)ULS	0.00	1.20	11.55	1.88	0.87	2.65	0.92	2.65	0.87
ARMA(1,1)ULS	-0.01	1.27	12.74	2.65	0.87	2.65	0.92	2.65	0.87
ARMA(2,2) ULS	0.11	6.79	1385.98	9.44	0.82	21.97	0.98	9.23	0.85
ARMA(2,2) MLE	0.03	2.38	222.47	3.65	0.84	6.60	0.99	3.58	0.84
Kalman Filter ARMA(2,2)	-0.01	1.29	13.31	1.74	0.83	2.65	0.93	3.33	0.89
MMAE ARMA(2,2)	0.01	1.26	12.77	2.01	0.91	2.76	0.90	3.02	0.88

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	50.1	49.4

Table 35. Monte Carlo Simulation Results for Case 14

TRUE PARAMETERS

Phi1 =	-0.7	Sample Size =	300
Phi2 =	-0.6	Number of Predictions=	5
Theta1=	-0.4	Noise Std. Deviation =	1
Theta2=	0.5	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.02	8.34	1.77	0.92	3.76	1.00	2.86	0.90
ARMA(2,2)InitialEstimates	0.00	1.14	10.36	1.83	0.89	4.04	1.00	2.84	0.87
Naive	0.03	2.02	33.23	4.59	0.91	2.64	0.92	4.52	0.91
Moving Average (T=5)	0.02	1.36	15.22	3.24	0.91	3.03	0.92	2.79	0.89
Moving Average (T=10)	0.00	1.32	14.18	2.97	0.92	2.89	0.92	2.80	0.90
Simple Average	0.01	1.30	13.67	2.86	0.92	2.84	0.91	2.82	0.88
Exponential Smoothing	0.03	1.63	21.63	3.46	0.88	2.42	0.95	3.15	0.83
Regression	0.01	1.31	13.70	2.83	0.92	2.83	0.91	2.83	0.90
AR(1)ULS	0.01	1.31	13.58	2.69	0.93	3.25	0.92	2.72	0.86
AR(2)ULS	0.00	1.13	10.06	1.86	0.90	4.07	0.99	3.57	0.98
MA(1)ULS	0.01	1.29	13.21	2.61	0.92	2.83	0.91	2.83	0.89
MA(2)ULS	0.01	1.24	12.56	2.04	0.92	2.83	0.91	2.83	0.89
ARMA(1,1)ULS	0.01	1.31	13.69	2.83	0.92	2.83	0.91	2.83	0.89
ARMA(2,2) ULS	0.00	1.03	8.70	1.73	0.92	3.32	0.99	2.80	0.90
ARMA(2,2) MLE	0.00	1.03	8.61	1.80	0.90	3.61	1.00	2.85	0.89
Kalman Filter ARMA(2,2)	0.00	1.23	12.74	1.73	0.89	2.66	0.91	3.52	0.89
MMAE ARMA(2,2)	0.00	1.27	14.44	1.80	0.93	2.35	0.84	2.67	0.80

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	37.3	37.4

Table 36. Monte Carlo Simulation Results for Case 15

TRUE PARAMETERS

Phi1 =	-0.7	Sample Size =	100
Phi2 =	-0.7	Number of Predictions=	5
Theta1=	-0.3	Noise Std. Deviation =	1
Theta2=	-0.3	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.89	6.07	1.65	0.90	2.17	0.96	1.98	0.91
ARMA(2,2)InitialEstimates	-0.01	1.09	8.99	2.18	0.90	2.69	0.94	1.98	0.87
Naive	0.00	1.39	14.47	3.23	0.89	2.15	0.87	3.05	0.92
Moving Average (T=5)	-0.03	1.00	7.86	2.24	0.89	2.15	0.93	1.99	0.86
Moving Average (T=10)	-0.02	0.98	7.26	2.07	0.88	2.04	0.91	1.99	0.90
Simple Average	-0.01	0.96	7.01	2.04	0.91	2.01	0.93	1.97	0.89
Exponential Smoothing	-0.01	1.15	9.98	2.39	0.84	1.87	0.90	2.19	0.86
Regression	-0.02	0.96	7.09	1.99	0.90	1.99	0.92	1.99	0.89
AR(1)ULS	-0.01	0.95	6.90	1.87	0.90	2.20	0.95	1.93	0.87
AR(2)ULS	-0.01	0.91	6.34	1.65	0.90	2.47	0.96	2.22	0.94
MA(1)ULS	-0.01	0.95	6.82	1.84	0.91	1.97	0.91	1.97	0.89
MA(2)ULS	-0.02	0.95	6.79	1.76	0.89	1.97	0.91	1.97	0.89
ARMA(1,1)ULS	-0.01	0.96	7.00	1.99	0.90	1.97	0.92	1.97	0.88
ARMA(2,2) ULS	-0.01	0.93	6.65	1.72	0.89	1.98	0.91	1.97	0.89
ARMA(2,2) MLE	-0.02	0.92	6.53	1.68	0.90	2.09	0.92	2.02	0.90
Kalman Filter ARMA(2,2)	-0.02	1.07	9.37	1.70	0.92	2.26	0.91	2.72	0.90
MMAE ARMA(2,2)	0.00	1.09	9.80	1.79	0.93	1.99	0.86	2.09	0.81

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	35.3	34.4

Table 37. Monte Carlo Simulation Results for Case 16

TRUE PARAMETERS

Phi1 =	1	Sample Size =	200
Phi2 =	-0.6	Number of Predictions=	5
Theta1=	0.4	Noise Std. Deviation =	1
Theta2=	0.2	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.02	1.01	8.01	1.76	0.86	2.41	0.93	2.33	0.91
ARMA(2,2)InitialEstimates	0.09	2.46	35.09	7.02	0.95	2.61	0.87	2.35	0.83
Naive	-0.04	1.66	21.81	2.45	0.90	4.08	0.91	3.42	0.88
Moving Average (T=5)	-0.01	1.29	12.86	2.84	0.84	2.54	0.86	2.39	0.90
Moving Average (T=10)	-0.08	1.20	11.19	2.49	0.85	2.44	0.86	2.33	0.90
Simple Average	-0.02	1.15	10.15	2.39	0.87	2.38	0.88	2.35	0.89
Exponential Smoothing	-0.01	1.55	18.71	2.58	0.91	3.46	0.87	2.78	0.86
Regression	-0.03	1.16	10.22	2.36	0.85	2.36	0.88	2.36	0.91
AR(1)ULS	-0.02	1.13	9.79	2.15	0.82	2.67	0.93	2.45	0.93
AR(2)ULS	-0.01	1.06	8.67	1.70	0.82	3.14	0.99	3.09	0.98
MA(1)ULS	-0.02	1.13	9.69	2.12	0.83	2.35	0.87	2.35	0.89
MA(2)ULS	-0.03	1.10	9.33	1.86	0.86	2.35	0.87	2.35	0.89
ARMA(1,1)ULS	0.02	1.21	11.26	2.39	0.84	2.52	0.87	2.34	0.90
ARMA(2,2) ULS	0.11	2.29	34.73	4.84	0.83	2.55	0.86	2.37	0.87
ARMA(2,2) MLE	-0.12	1.12	12.11	2.44	0.89	2.83	0.93	2.52	0.90
Kalman Filter ARMA(2,2)	-0.02	1.08	9.01	1.72	0.84	2.37	0.88	2.38	0.89
MMAE ARMA(2,2)	-0.02	1.09	9.12	1.82	0.86	2.12	0.85	2.40	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	43.4	40.6

Table 38. Monte Carlo Simulation Results for Case 17

TRUE PARAMETERS

Phi1 =	0.6	Sample Size =	500
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	-0.3	Noise Std. Deviation =	1
Theta2=	0.3	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.01	1.07	9.27	1.69	0.90	2.56	0.95	2.54	0.91
ARMA(2,2)InitialEstimates	0.01	1.11	9.88	1.84	0.91	2.55	0.96	2.54	0.88
Naive	0.02	1.86	28.18	2.78	0.90	4.29	0.86	3.26	0.88
Moving Average (T=5)	0.02	1.42	15.92	2.88	0.85	2.66	0.89	2.73	0.91
Moving Average (T=10)	0.01	1.36	14.74	2.69	0.88	2.58	0.86	2.62	0.90
Simple Average	0.00	1.27	13.11	2.56	0.88	2.55	0.87	2.55	0.90
Exponential Smoothing	0.01	1.61	20.66	2.82	0.87	3.19	0.83	2.64	0.88
Regression	-0.01	1.27	13.13	2.54	0.88	2.54	0.88	2.54	0.91
AR(1)ULS	0.00	1.26	12.78	2.36	0.89	2.85	0.89	2.51	0.88
AR(2)ULS	0.01	1.09	9.44	1.76	0.90	3.60	1.00	3.06	0.94
MA(1)ULS	0.00	1.25	12.52	2.31	0.88	2.54	0.87	2.54	0.90
MA(2)ULS	0.01	1.20	11.49	1.85	0.89	2.54	0.87	2.54	0.90
ARMA(1,1)ULS	0.00	1.27	13.16	2.57	0.88	2.54	0.87	2.54	0.90
ARMA(2,2) ULS	0.01	1.10	9.73	1.69	0.88	2.55	0.93	2.54	0.90
ARMA(2,2) MLE	0.01	1.08	9.39	1.69	0.87	2.56	0.95	2.54	0.91
Kalman Filter ARMA(2,2)	0.01	1.19	11.35	1.69	0.88	2.57	0.87	2.55	0.90
MMAE ARMA(2,2)	0.01	1.20	11.53	1.81	0.92	2.38	0.84	2.60	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	29.7	35.1

Table 39. Monte Carlo Simulation Results for Case 18

TRUE PARAMETERS

Phi1 =	0.6	Sample Size =	300
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	-0.3	Noise Std. Deviation =	1
Theta2=	0.3	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.06	1.08	9.10	1.70	0.90	2.57	0.96	2.55	0.91
ARMA(2,2)InitialEstimates	-0.06	1.14	9.84	1.87	0.91	2.56	0.95	2.55	0.92
Naive	0.04	1.84	27.67	2.78	0.94	4.32	0.86	3.28	0.91
Moving Average (T=5)	-0.08	1.34	13.88	2.89	0.90	2.67	0.88	2.73	0.92
Moving Average (T=10)	-0.09	1.28	12.79	2.70	0.89	2.59	0.88	2.62	0.92
Simple Average	-0.06	1.23	11.99	2.58	0.87	2.56	0.91	2.57	0.90
Exponential Smoothing	-0.01	1.56	19.99	2.84	0.91	3.22	0.78	2.66	0.90
Regression	-0.05	1.23	12.02	2.55	0.87	2.55	0.90	2.55	0.89
AR(1)ULS	-0.06	1.22	11.67	2.37	0.88	2.86	0.94	2.52	0.91
AR(2)ULS	-0.06	1.08	9.03	1.75	0.90	3.61	0.99	3.09	0.97
MA(1)ULS	-0.06	1.21	11.45	2.31	0.87	2.55	0.89	2.55	0.91
MA(2)ULS	-0.05	1.16	10.74	1.86	0.89	2.55	0.89	2.55	0.91
ARMA(1,1)ULS	-0.06	1.24	12.08	2.58	0.87	2.55	0.90	2.55	0.91
ARMA(2,2) ULS	-0.07	1.09	9.43	1.71	0.89	2.56	0.95	2.55	0.90
ARMA(2,2) MLE	-0.06	1.10	9.28	1.69	0.89	2.58	0.95	2.55	0.91
Kalman Filter ARMA(2,2)	-0.06	1.16	10.63	1.70	0.89	2.59	0.91	2.56	0.91
MMAE ARMA(2,2)	-0.06	1.16	10.66	1.82	0.91	2.36	0.84	2.60	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	43.6	36.7

Table 40. Monte Carlo Simulation Results for Case 19

TRUE PARAMETERS

Phi1 =	0.6	Sample Size =	600
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	-0.6	Noise Std. Deviation =	1
Theta2=	-0.5	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.05	1.20	11.34	1.68	0.92	3.02	0.93	2.97	0.93
ARMA(2,2)InitialEstimates	0.03	1.43	16.07	2.49	0.87	3.15	0.90	2.97	0.91
Naive	0.15	1.94	31.35	2.52	0.86	4.79	0.90	4.14	0.89
Moving Average (T=5)	0.10	1.67	21.79	3.35	0.89	3.47	0.88	3.40	0.92
Moving Average (T=10)	0.08	1.56	18.56	3.13	0.91	3.23	0.92	3.19	0.92
Simple Average	0.06	1.42	15.55	2.99	0.91	3.00	0.91	3.00	0.91
Exponential Smoothing	0.14	1.87	28.83	2.85	0.91	4.27	0.88	3.68	0.86
Regression	0.04	1.44	16.01	2.97	0.90	2.97	0.91	2.97	0.90
AR(1)ULS	0.06	1.39	14.94	2.58	0.89	3.25	0.91	3.03	0.93
AR(2)ULS	0.04	1.26	12.60	1.77	0.87	4.13	0.99	3.83	0.99
MA(1)ULS	0.06	1.39	14.85	2.57	0.89	2.97	0.91	2.97	0.91
MA(2)ULS	0.07	1.31	13.59	1.80	0.90	2.97	0.91	2.97	0.91
ARMA(1,1)ULS	0.06	1.42	15.51	2.92	0.88	2.97	0.91	2.97	0.91
ARMA(2,2) ULS	0.05	1.22	11.79	1.69	0.88	3.00	0.91	2.97	0.91
ARMA(2,2) MLE	0.04	1.22	11.74	1.69	0.91	3.02	0.91	2.97	0.91
Kalman Filter ARMA(2,2)	0.05	1.29	13.30	1.70	0.91	2.98	0.91	2.98	0.91
MMAE ARMA(2,2)	0.04	1.30	13.42	1.86	0.92	2.97	0.90	3.11	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	41.1	37.1

Table 41. Monte Carlo Simulation Results for Case 20

TRUE PARAMETERS

Phi1 =	0.6	Sample Size =	400
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	-0.6	Noise Std. Deviation =	1
Theta2=	-0.5	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	1.26	12.73	1.68	0.90	3.01	0.91	2.96	0.93
ARMA(2,2)InitialEstimates	-0.01	1.48	17.05	2.50	0.89	3.14	0.86	2.96	0.91
Naive	-0.07	1.88	29.42	2.52	0.90	4.79	0.93	4.13	0.91
Moving Average (T=5)	0.00	1.59	20.31	3.34	0.91	3.45	0.91	3.38	0.92
Moving Average (T=10)	-0.06	1.48	17.76	3.12	0.91	3.22	0.91	3.17	0.93
Simple Average	-0.01	1.40	15.16	2.99	0.92	3.01	0.89	3.00	0.92
Exponential Smoothing	-0.07	1.80	27.07	2.85	0.94	4.27	0.90	3.67	0.89
Regression	0.00	1.40	15.22	2.96	0.91	2.96	0.90	2.96	0.92
AR(1)ULS	-0.02	1.36	14.42	2.57	0.94	3.25	0.93	3.03	0.92
AR(2)ULS	0.00	1.32	13.68	1.76	0.90	4.13	0.97	3.83	0.99
MA(1)ULS	-0.01	1.36	14.43	2.57	0.93	2.96	0.88	2.96	0.92
MA(2)ULS	-0.02	1.31	13.55	1.80	0.90	2.96	0.88	2.96	0.92
ARMA(1,1)ULS	0.00	1.39	15.02	2.90	0.91	2.97	0.88	2.96	0.92
ARMA(2,2) ULS	-0.01	1.26	12.61	1.70	0.90	2.98	0.91	2.96	0.93
ARMA(2,2) MLE	0.00	1.26	12.74	1.69	0.91	3.00	0.91	2.96	0.93
Kalman Filter ARMA(2,2)	-0.01	1.27	13.04	1.70	0.91	2.98	0.88	2.97	0.92
MMAE ARMA(2,2)	0.00	1.28	13.18	1.86	0.94	2.96	0.89	3.10	0.94

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	46.6	48

Table 42. Monte Carlo Simulation Results for Case 21

TRUE PARAMETERS

Phi1 =	0.6	Sample Size =	200
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	-0.6	Noise Std. Deviation =	1
Theta2=	-0.5	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.02	1.26	12.98	1.70	0.84	3.02	0.89	2.97	0.93
ARMA(2,2)InitialEstimates	0.00	1.46	16.85	2.53	0.92	3.14	0.89	2.96	0.90
Naive	-0.01	1.86	27.28	2.54	0.87	4.81	0.94	4.13	0.90
Moving Average (T=5)	-0.05	1.69	21.88	3.36	0.87	3.46	0.86	3.39	0.93
Moving Average (T=10)	-0.03	1.56	18.72	3.13	0.86	3.22	0.88	3.18	0.94
Simple Average	-0.03	1.45	16.37	3.01	0.87	3.04	0.87	3.03	0.95
Exponential Smoothing	-0.03	1.79	25.08	2.87	0.91	4.29	0.89	3.67	0.90
Regression	-0.03	1.45	16.53	2.96	0.88	2.96	0.86	2.96	0.95
AR(1)ULS	-0.03	1.41	15.43	2.58	0.87	3.26	0.90	3.03	0.95
AR(2)ULS	0.00	1.32	14.49	1.76	0.85	4.14	0.97	3.83	0.96
MA(1)ULS	-0.03	1.41	15.47	2.57	0.87	2.96	0.87	2.96	0.95
MA(2)ULS	-0.02	1.33	14.09	1.81	0.82	2.96	0.87	2.96	0.95
ARMA(1,1)ULS	0.01	1.45	16.36	2.90	0.88	2.99	0.85	2.97	0.96
ARMA(2,2) ULS	-0.02	1.28	13.19	1.74	0.80	2.98	0.87	2.96	0.95
ARMA(2,2) MLE	-0.02	1.29	13.39	1.71	0.84	3.05	0.89	2.97	0.91
Kalman Filter ARMA(2,2)	-0.01	1.31	13.63	1.72	0.85	3.01	0.87	2.99	0.95
MMAE ARMA(2,2)	-0.02	1.33	13.87	1.87	0.84	2.95	0.87	3.10	0.96

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	46.6	46.4

Table 43. Monte Carlo Simulation Results for Case 22

TRUE PARAMETERS

Phi1 =	0.3	Sample Size =	200
Phi2 =	-0.3	Number of Predictions=	5
Theta1=	1.4	Noise Std. Deviation =	1
Theta2=	-0.7	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.17	10.97	1.90	0.86	2.55	0.95	2.55	0.88
ARMA(2,2)InitialEstimates	0.31	19.46	41.19	36.09	0.80	70.97	1.00	25.05	0.80
Naive	-0.02	1.84	26.69	4.37	0.90	3.43	0.91	3.60	0.88
Moving Average (T=5)	-0.01	1.30	13.15	2.80	0.89	2.57	0.95	2.61	0.88
Moving Average (T=10)	0.02	1.28	12.74	2.68	0.89	2.55	0.95	2.57	0.88
Simple Average	0.00	1.27	12.45	2.59	0.89	2.55	0.93	2.55	0.89
Exponential Smoothing	-0.01	1.52	17.83	3.15	0.84	2.54	0.92	2.69	0.84
Regression	0.00	1.27	12.49	2.56	0.87	2.56	0.94	2.56	0.88
AR(1)ULS	0.00	1.25	12.15	2.32	0.86	2.67	0.95	2.61	0.89
AR(2)ULS	0.00	1.27	12.57	1.95	0.86	3.22	0.99	2.98	0.92
MA(1)ULS	0.00	1.24	12.05	2.29	0.87	2.54	0.94	2.54	0.89
MA(2)ULS	0.00	1.21	11.53	1.82	0.83	2.54	0.94	2.54	0.89
ARMA(1,1)ULS	0.00	1.26	12.38	2.47	0.85	2.55	0.93	2.55	0.89
ARMA(2,2) ULS	0.31	19.19	40.97	35.62	0.81	70.08	0.99	24.75	0.80
ARMA(2,2) MLE	-0.02	2.07	88.31	4.66	0.88	6.03	0.95	2.96	0.87
Kalman Filter ARMA(2,2)	0.00	1.22	11.91	1.79	0.84	2.62	0.91	2.58	0.89
MMAE ARMA(2,2)	0.00	1.23	12.16	2.14	0.91	3.24	0.94	3.36	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	59.4	58.5

Table 44. Monte Carlo Simulation Results for Case 23

TRUE PARAMETERS

Phi1 =	0.4	Sample Size =	300
Phi2 =	-0.3	Number of Predictions=	5
Theta1=	1	Noise Std. Deviation =	1
Theta2=	-0.6	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.89	6.21	1.71	0.89	1.97	0.91	1.97	0.91
ARMA(2,2)InitialEstimates	0.00	1.33	14.72	3.61	0.89	2.72	0.90	2.01	0.90
Naive	0.02	1.39	15.90	3.33	0.88	2.64	0.91	2.79	0.91
Moving Average (T=5)	-0.01	0.93	7.23	2.16	0.89	2.03	0.91	2.05	0.92
Moving Average (T=10)	-0.02	0.92	7.04	2.07	0.88	2.00	0.90	2.01	0.90
Simple Average	-0.01	0.90	6.71	1.99	0.89	1.97	0.90	1.98	0.91
Exponential Smoothing	0.01	1.10	10.29	2.39	0.85	1.99	0.87	2.11	0.87
Regression	0.00	0.90	6.73	1.97	0.88	1.97	0.90	1.97	0.91
AR(1)ULS	-0.01	0.88	6.40	1.82	0.93	2.07	0.90	2.02	0.91
AR(2)ULS	-0.01	0.89	6.30	1.69	0.88	2.33	0.95	2.19	0.94
MA(1)ULS	-0.01	0.89	6.40	1.80	0.92	1.97	0.90	1.97	0.91
MA(2)ULS	-0.01	0.88	6.23	1.70	0.90	1.97	0.90	1.97	0.91
ARMA(1,1)ULS	-0.01	0.90	6.65	1.95	0.91	1.97	0.90	1.97	0.91
ARMA(2,2) ULS	-0.01	0.88	6.20	1.68	0.90	1.97	0.91	1.97	0.91
ARMA(2,2) MLE	-0.01	0.89	6.28	1.69	0.91	1.98	0.91	1.97	0.91
Kalman Filter ARMA(2,2)	0.00	0.93	7.00	1.68	0.91	2.19	0.91	2.34	0.90
MMAE ARMA(2,2)	0.00	0.93	6.96	1.89	0.96	2.16	0.89	2.19	0.92

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	49.9	46

Table 45. Monte Carlo Simulation Results for Case 24

TRUE PARAMETERS

Phi1 =	0.4	Sample Size =	300
Phi2 =	0.4	Number of Predictions=	5
Theta1=	0.2	Noise Std. Deviation =	5
Theta2=	0.2	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.23	4.29	139.89	8.43	0.88	8.96	0.91	9.14	0.91
ARMA(2,2)InitialEstimates	-0.59	8.72	649.45	20.31	0.88	13.69	0.88	10.33	0.84
Naive	-0.07	5.48	250.41	10.90	0.87	11.35	0.90	11.79	0.89
Moving Average (T=5)	-0.36	4.57	158.19	8.84	0.85	9.74	0.92	10.17	0.90
Moving Average (T=10)	-0.32	4.61	167.58	8.98	0.84	9.66	0.91	9.97	0.89
Simple Average	-0.17	4.67	168.07	9.41	0.87	9.52	0.93	9.59	0.88
Exponential Smoothing	-0.19	4.82	181.74	8.81	0.84	9.47	0.87	9.97	0.87
Regression	-0.07	4.73	174.48	9.22	0.87	9.22	0.90	9.22	0.88
AR(1)ULS	-0.16	4.59	162.83	8.87	0.88	9.01	0.90	9.14	0.87
AR(2)ULS	-0.18	4.37	146.47	8.45	0.92	8.91	0.93	9.23	0.89
MA(1)ULS	-0.16	4.64	166.19	9.02	0.87	9.32	0.92	9.31	0.87
MA(2)ULS	0.49	40.80	832.38	18.02	0.53	9.32	0.92	9.31	0.87
ARMA(1,1)ULS	-0.15	4.59	162.65	9.01	0.85	9.25	0.92	9.30	0.87
ARMA(2,2) ULS	-0.15	4.81	185.71	11.25	0.91	10.00	0.94	9.84	0.89
ARMA(2,2) MLE	-0.20	4.31	142.63	8.81	0.89	9.34	0.92	9.46	0.91
Kalman Filter ARMA(2,2)	-0.08	4.40	148.73	8.61	0.87	9.07	0.93	9.26	0.90
MMAE ARMA(2,2)	-0.15	4.38	148.23	8.41	0.88	8.78	0.91	8.95	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	50.1	49.3

Table 46. Monte Carlo Simulation Results for Case 25

TRUE PARAMETERS

Phi1 =	0.4	Sample Size =	300
Phi2 =	0.3	Number of Predictions=	5
Theta1=	-0.4	Noise Std. Deviation =	5
Theta2=	0.3	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.34	5.22	211.08	8.60	0.90	11.29	0.89	11.50	0.91
ARMA(2,2)InitialEstimates	-0.24	6.47	328.76	10.98	0.89	12.95	0.88	11.89	0.89
Naive	-0.24	6.29	316.09	9.68	0.91	13.36	0.90	14.74	0.92
Moving Average (T=5)	-0.57	6.09	280.36	10.70	0.87	13.04	0.93	13.70	0.94
Moving Average (T=10)	-0.50	6.17	295.83	11.28	0.88	12.76	0.88	13.20	0.91
Simple Average	-0.30	5.89	269.56	11.73	0.89	11.99	0.90	12.08	0.90
Exponential Smoothing	-0.34	6.04	285.92	9.51	0.89	12.46	0.91	13.70	0.88
Regression	-0.16	6.05	282.02	11.51	0.87	11.51	0.89	11.51	0.86
AR(1)ULS	-0.29	5.68	249.66	10.05	0.89	11.01	0.88	11.41	0.89
AR(2)ULS	-0.30	5.26	216.70	8.74	0.89	11.52	0.90	12.57	0.92
MA(1)ULS	-0.29	5.73	254.64	10.21	0.89	11.63	0.90	11.63	0.89
MA(2)ULS	2.32	43.42	1030.43	22.70	0.72	11.63	0.90	11.63	0.89
ARMA(1,1)ULS	-0.27	5.82	262.88	11.25	0.89	11.59	0.90	11.62	0.89
ARMA(2,2) ULS	-0.24	5.40	236.57	8.97	0.88	11.41	0.90	11.56	0.89
ARMA(2,2) MLE	-0.29	5.20	209.52	8.85	0.89	11.32	0.91	11.55	0.92
Kalman Filter ARMA(2,2)	-0.56	5.31	220.17	9.32	0.91	11.19	0.87	11.64	0.91
MMAE ARMA(2,2)	-0.47	5.25	215.61	9.01	0.90	10.57	0.86	11.15	0.89

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	48.8	48.1

Table 47. Monte Carlo Simulation Results for Case 26

TRUE PARAMETERS

Phi1 =	0.5	Sample Size =	300
Phi2 =	0.3	Number of Predictions=	5
Theta1=	-0.4	Noise Std. Deviation =	5
Theta2=	-0.5	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.23	7.09	414.30	8.67	0.89	16.88	0.90	18.66	0.88
ARMA(2,2)InitialEstimates	-0.35	9.04	659.53	15.41	0.87	18.28	0.91	19.29	0.87
Naive	-0.57	8.30	546.78	9.76	0.81	16.84	0.86	20.91	0.91
Moving Average (T=5)	-0.74	9.76	737.10	14.00	0.85	20.39	0.89	22.66	0.91
Moving Average (T=10)	-0.50	10.39	864.57	16.94	0.85	21.14	0.90	22.76	0.91
Simple Average	-0.23	11.00	898.34	20.43	0.91	21.11	0.89	21.43	0.89
Exponential Smoothing	-0.62	8.33	547.08	10.33	0.84	17.05	0.86	20.77	0.91
Regression	0.03	11.30	981.77	19.77	0.90	19.77	0.87	19.77	0.88
AR(1)ULS	-0.25	10.41	817.56	16.19	0.90	17.52	0.79	18.53	0.86
AR(2)ULS	-0.75	7.75	481.32	9.43	0.78	16.22	0.85	20.05	0.90
MA(1)ULS	-0.24	10.62	846.73	16.93	0.90	20.25	0.88	20.25	0.88
MA(2)ULS	0.07	69.72	406.49	35.18	0.66	20.25	0.88	20.25	0.88
ARMA(1,1)ULS	0.05	10.91	884.82	18.97	0.91	20.04	0.88	20.25	0.88
ARMA(2,2) ULS	0.47	11.13	1212.19	21.18	0.89	21.17	0.88	21.11	0.87
ARMA(2,2) MLE	-0.33	7.22	435.09	9.06	0.88	16.76	0.89	18.55	0.87
Kalman Filter ARMA(2,2)	-0.49	7.54	485.21	9.59	0.85	15.83	0.84	18.28	0.86
MMAE ARMA(2,2)	-0.33	7.77	506.68	8.94	0.83	12.99	0.85	14.99	0.84

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	47.2	43.5

Table 48. Monte Carlo Simulation Results for Case 27

TRUE PARAMETERS

Phi1 =	0.7	Sample Size =	300
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0.8	Noise Std. Deviation =	5
Theta2=	-0.1	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.19	4.09	131.92	8.58	0.89	8.77	0.92	8.89	0.91
ARMA(2,2)InitialEstimates	-0.35	9.75	892.49	30.46	0.84	11.30	0.82	9.41	0.85
Naive	0.02	5.75	277.94	12.84	0.88	11.58	0.92	11.69	0.90
Moving Average (T=5)	-0.25	4.25	142.16	9.00	0.86	9.10	0.94	9.35	0.90
Moving Average (T=10)	-0.22	4.28	146.34	8.85	0.86	9.04	0.91	9.24	0.88
Simple Average	-0.11	4.51	157.67	9.19	0.89	9.22	0.93	9.27	0.91
Exponential Smoothing	-0.09	4.76	187.29	9.52	0.84	9.12	0.86	9.33	0.87
Regression	-0.04	4.56	162.06	9.00	0.89	9.00	0.91	9.00	0.91
AR(1)ULS	-0.09	4.51	157.51	9.17	0.89	9.23	0.93	9.25	0.89
AR(2)ULS	-0.11	4.36	147.78	8.68	0.92	8.94	0.91	9.05	0.89
MA(1)ULS	-0.11	4.56	160.69	9.32	0.89	9.09	0.92	9.09	0.89
MA(2)ULS	5.35	22.68	662.56	85.15	0.73	9.09	0.92	9.09	0.89
ARMA(1,1)ULS	3.91	53.00	602.03	80.57	0.85	35.31	0.88	19.40	0.87
ARMA(2,2) ULS	-0.22	6.88	461.65	20.32	0.91	12.07	0.92	10.68	0.92
ARMA(2,2) MLE	-0.44	5.15	406.26	11.96	0.92	11.12	0.92	10.19	0.88
Kalman Filter ARMA(2,2)	-0.10	4.32	146.89	8.74	0.90	8.89	0.92	9.23	0.90
MMAE ARMA(2,2)	-0.10	4.36	150.96	8.58	0.89	8.67	0.92	8.72	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	42.5	42.3

Table 49. Monte Carlo Simulation Results for Case 28

TRUE PARAMETERS

Phi1 =	-0.4	Sample Size =	400
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0.5	Noise Std. Deviation =	5
Theta2=	0.3	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.03	5.39	227.50	8.74	0.87	12.00	0.94	11.85	0.87
ARMA(2,2)InitialEstimates	0.05	6.99	377.27	12.85	0.88	13.05	0.81	12.17	0.90
Naive	0.31	9.14	667.00	21.39	0.87	18.69	0.87	17.59	0.87
Moving Average (T=5)	0.03	6.37	314.80	13.24	0.86	11.85	0.84	12.00	0.88
Moving Average (T=10)	0.05	6.25	299.75	12.47	0.87	11.81	0.84	11.87	0.90
Simple Average	0.04	6.12	290.21	11.96	0.86	11.83	0.90	11.84	0.90
Exponential Smoothing	0.19	7.45	434.25	14.61	0.80	13.23	0.84	12.77	0.84
Regression	0.05	6.12	290.84	11.87	0.85	11.87	0.90	11.87	0.90
AR(1)ULS	0.03	5.90	269.59	10.30	0.86	11.47	0.86	11.87	0.90
AR(2)ULS	-0.02	6.35	321.87	10.72	0.91	13.94	0.91	15.05	0.90
MA(1)ULS	0.03	5.95	274.41	10.41	0.86	11.84	0.90	11.83	0.90
MA(2)ULS	0.55	14.30	481.16	38.50	0.43	11.84	0.90	11.83	0.90
ARMA(1,1)ULS	0.04	6.11	289.77	11.79	0.86	11.84	0.90	11.84	0.90
ARMA(2,2) ULS	-0.57	27.34	521.88	31.19	0.84	37.75	0.91	28.26	0.94
ARMA(2,2) MLE	0.04	5.45	231.85	8.80	0.89	12.09	0.94	11.92	0.86
Kalman Filter ARMA(2,2)	0.09	6.53	430.00	9.73	0.86	12.81	0.91	18.25	0.87
MMAE ARMA(2,2)	0.11	6.71	482.38	9.11	0.85	10.39	0.84	10.76	0.72

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	36.7	39.8

Table 50. Monte Carlo Simulation Results for Case 29

TRUE PARAMETERS

Phi1 =	-0.2	Sample Size =	300
Phi2 =	0.1	Number of Predictions=	5
Theta1=	-0.4	Noise Std. Deviation =	5
Theta2=	0.5	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.15	4.22	140.46	8.72	0.88	9.40	0.94	9.36	0.90
ARMA(2,2)InitialEstimates	-0.10	4.76	180.60	10.07	0.87	9.79	0.93	9.38	0.90
Naive	0.12	6.61	355.61	12.98	0.89	12.86	0.92	13.12	0.88
Moving Average (T=5)	-0.22	4.71	174.12	10.42	0.90	10.07	0.95	10.04	0.93
Moving Average (T=10)	-0.26	4.51	161.08	9.87	0.89	9.69	0.94	9.66	0.91
Simple Average	-0.15	4.38	150.90	9.47	0.92	9.43	0.94	9.43	0.90
Exponential Smoothing	-0.02	5.51	243.41	10.77	0.89	10.21	0.89	10.30	0.87
Regression	-0.10	4.39	151.19	9.37	0.92	9.37	0.93	9.37	0.91
AR(1)ULS	-0.13	4.39	152.30	9.52	0.91	9.60	0.93	9.61	0.91
AR(2)ULS	-0.15	4.41	150.38	8.85	0.89	9.70	0.94	9.76	0.91
MA(1)ULS	-0.15	4.37	150.51	9.35	0.91	9.36	0.93	9.36	0.90
MA(2)ULS	-0.14	4.36	150.88	8.59	0.89	9.36	0.93	9.36	0.90
ARMA(1,1)ULS	-0.14	4.38	150.93	9.37	0.93	9.36	0.94	9.36	0.90
ARMA(2,2) ULS	-0.15	4.24	142.06	8.57	0.88	9.40	0.94	9.36	0.90
ARMA(2,2) MLE	-0.09	4.28	146.93	9.30	0.90	9.56	0.94	9.43	0.90
Kalman Filter ARMA(2,2)	-0.15	4.42	152.49	9.27	0.91	9.45	0.92	9.55	0.92
MMAE ARMA(2,2)	-0.14	4.42	152.41	9.13	0.91	9.17	0.90	9.20	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	40.7	43

Table 51. Monte Carlo Simulation Results for Case 30

TRUE PARAMETERS

Phi1 =	-0.2	Sample Size =	200
Phi2 =	0.6	Number of Predictions=	5
Theta1=	-0.5	Noise Std. Deviation =	5
Theta2=	-0.5	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.02	5.41	232.46	8.67	0.85	12.56	0.89	13.46	0.94
ARMA(2,2)InitialEstimates	-0.02	6.57	338.30	13.24	0.93	13.01	0.86	13.66	0.94
Naive	-0.50	8.14	542.09	19.70	0.93	21.36	0.87	21.68	0.90
Moving Average (T=5)	-0.17	7.38	420.44	14.40	0.94	15.15	0.88	15.84	0.93
Moving Average (T=10)	0.37	7.20	403.59	14.19	0.92	15.16	0.92	15.53	0.94
Simple Average	-0.03	7.01	388.22	14.29	0.89	14.40	0.92	14.50	0.95
Exponential Smoothing	-0.31	7.34	421.75	14.15	0.91	16.23	0.85	16.92	0.90
Regression	-0.06	6.96	392.31	14.00	0.91	14.00	0.90	14.00	0.92
AR(1)ULS	-0.07	6.98	382.80	13.98	0.89	14.08	0.89	14.14	0.93
AR(2)ULS	-0.01	5.73	257.36	9.46	0.85	13.10	0.90	14.78	0.95
MA(1)ULS	-0.05	7.07	395.03	14.57	0.89	14.06	0.90	14.05	0.92
MA(2)ULS	-0.84	11.12	5281.90	34.75	0.84	14.06	0.90	14.05	0.92
ARMA(1,1)ULS	-0.03	7.00	387.44	14.03	0.88	14.06	0.90	14.04	0.92
ARMA(2,2) ULS	0.02	5.56	243.35	8.55	0.84	12.85	0.88	13.59	0.94
ARMA(2,2) MLE	0.03	5.59	245.69	8.58	0.84	12.83	0.88	13.57	0.94
Kalman Filter ARMA(2,2)	0.00	6.88	372.50	13.19	0.88	14.36	0.88	15.74	0.93
MMAE ARMA(2,2)	-0.02	6.84	371.78	12.47	0.84	12.78	0.84	12.94	0.86

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	30.5	28.9

Table 52. Monte Carlo Simulation Results for Case 31

TRUE PARAMETERS

Phi1 =	-0.7	Sample Size =	300
Phi2 =	-0.6	Number of Predictions=	5
Theta1=	-0.4	Noise Std. Deviation =	5
Theta2=	0.5	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.02	5.22	217.17	9.04	0.90	18.93	1.00	14.39	0.92
ARMA(2,2)InitialEstimates	0.03	5.73	261.54	9.29	0.89	20.15	0.99	14.29	0.87
Naive	0.15	10.13	838.28	23.04	0.91	13.35	0.92	22.68	0.91
Moving Average (T=5)	0.05	6.86	386.51	16.26	0.92	15.26	0.92	14.02	0.87
Moving Average (T=10)	-0.03	6.69	359.62	14.92	0.92	14.54	0.91	14.06	0.89
Simple Average	0.04	6.59	347.06	14.39	0.93	14.27	0.91	14.17	0.89
Exponential Smoothing	0.12	8.17	546.33	17.40	0.88	12.22	0.94	15.83	0.83
Regression	0.06	6.60	347.82	14.24	0.92	14.24	0.91	14.24	0.90
AR(1)ULS	0.04	6.60	344.85	13.53	0.94	16.31	0.92	13.66	0.86
AR(2)ULS	-0.01	5.64	255.03	10.01	0.92	21.48	0.99	18.27	0.98
MA(1)ULS	0.04	6.51	335.62	13.12	0.94	14.20	0.91	14.20	0.89
MA(2)ULS	-1.03	13.14	5286.11	36.31	0.65	14.20	0.91	14.20	0.89
ARMA(1,1)ULS	0.04	6.59	347.19	14.20	0.93	14.20	0.91	14.20	0.89
ARMA(2,2) ULS	-0.11	6.07	491.85	10.47	0.89	17.74	0.97	14.47	0.91
ARMA(2,2) MLE	0.02	5.19	215.47	8.85	0.90	18.16	1.00	14.36	0.91
Kalman Filter ARMA(2,2)	0.08	6.84	394.77	13.24	0.92	14.36	0.93	16.84	0.85
MMAE ARMA(2,2)	0.05	6.82	386.63	12.08	0.90	12.13	0.85	12.18	0.74

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	31.9	29.5

Table 53. Monte Carlo Simulation Results for Case 32

TRUE PARAMETERS

Phi1 =	-0.7	Sample Size =	100
Phi2 =	-0.7	Number of Predictions=	5
Theta1=	-0.3	Noise Std. Deviation =	5
Theta2=	-0.3	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.07	4.49	155.81	8.37	0.91	10.91	0.95	9.99	0.90
ARMA(2,2)InitialEstimates	-0.04	5.47	228.90	11.01	0.90	13.37	0.94	9.98	0.86
Naive	0.09	7.01	370.15	16.25	0.90	10.96	0.88	15.38	0.92
Moving Average (T=5)	-0.14	5.06	200.55	11.30	0.90	10.86	0.93	10.08	0.86
Moving Average (T=10)	-0.08	4.92	185.00	10.47	0.89	10.32	0.91	10.05	0.86
Simple Average	-0.06	4.84	178.93	10.29	0.91	10.14	0.94	9.98	0.89
Exponential Smoothing	-0.02	5.78	254.35	12.05	0.83	9.50	0.90	11.05	0.86
Regression	-0.06	4.85	180.62	10.03	0.92	10.03	0.93	10.03	0.89
AR(1)ULS	-0.07	4.80	176.04	9.43	0.88	11.07	0.94	9.75	0.88
AR(2)ULS	-0.07	4.60	162.67	8.42	0.89	12.38	0.96	11.15	0.94
MA(1)ULS	-0.06	4.77	174.16	9.31	0.90	9.95	0.92	9.95	0.90
MA(2)ULS	-0.08	4.78	173.05	8.93	0.88	9.95	0.92	9.95	0.90
ARMA(1,1)ULS	-0.06	4.84	178.94	9.93	0.90	9.94	0.92	9.95	0.90
ARMA(2,2) ULS	-0.09	4.58	162.93	8.27	0.89	10.74	0.94	9.97	0.90
ARMA(2,2) MLE	-0.08	4.63	169.73	8.77	0.91	11.37	0.97	10.52	0.90
Kalman Filter ARMA(2,2)	-0.07	5.10	203.11	9.57	0.89	10.25	0.92	11.56	0.88
MMAE ARMA(2,2)	-0.09	5.10	201.33	8.78	0.88	8.90	0.83	8.97	0.75

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	35.2	33.6

Table 54. Monte Carlo Simulation Results for Case 33

TRUE PARAMETERS

Phi1 =	1	Sample Size =	200
Phi2 =	-0.6	Number of Predictions=	5
Theta1=	0.4	Noise Std. Deviation =	5
Theta2=	0.2	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.06	5.10	205.29	8.55	0.83	12.06	0.91	11.74	0.91
ARMA(2,2)InitialEstimates	0.38	11.42	493.41	25.32	0.87	12.94	0.84	11.76	0.81
Naive	-0.20	8.32	547.28	12.40	0.89	20.53	0.89	17.20	0.88
Moving Average (T=5)	-0.05	6.51	324.87	14.31	0.83	12.81	0.87	12.04	0.91
Moving Average (T=10)	-0.39	6.04	283.11	12.54	0.85	12.28	0.87	11.74	0.91
Simple Average	-0.09	5.80	256.79	12.02	0.86	11.99	0.91	11.85	0.90
Exponential Smoothing	-0.07	7.71	467.28	13.01	0.90	17.32	0.87	13.96	0.86
Regression	-0.11	5.82	258.56	11.87	0.83	11.87	0.87	11.87	0.91
AR(1)ULS	-0.10	5.70	247.54	10.86	0.84	13.46	0.93	12.32	0.92
AR(2)ULS	-0.05	5.35	221.57	8.72	0.81	15.67	0.99	15.45	0.97
MA(1)ULS	-0.09	5.67	245.13	10.70	0.83	11.83	0.89	11.84	0.91
MA(2)ULS	0.95	8.33	2350.14	29.41	0.87	11.83	0.89	11.84	0.91
ARMA(1,1)ULS	-0.08	5.80	256.87	11.83	0.84	11.85	0.89	11.84	0.91
ARMA(2,2) ULS	-0.04	10.55	493.12	20.80	0.82	13.44	0.86	12.08	0.85
ARMA(2,2) MLE	0.35	10.20	477.89	19.45	0.81	13.40	0.87	11.89	0.86
Kalman Filter ARMA(2,2)	-0.12	5.68	248.92	10.88	0.86	12.04	0.90	11.95	0.91
MMAE ARMA(2,2)	-0.12	5.72	250.87	10.04	0.79	10.19	0.83	10.29	0.85

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	42.6	41.8

Table 55. Monte Carlo Simulation Results for Case 34

TRUE PARAMETERS

Phi1 =	0.6	Sample Size =	300
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	-0.3	Noise Std. Deviation =	5
Theta2=	0.3	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.32	5.49	231.81	8.84	0.89	12.94	0.96	12.82	0.92
ARMA(2,2)InitialEstimates	-0.32	5.80	257.94	9.69	0.86	12.88	0.96	12.83	0.94
Naive	0.21	9.18	689.19	14.07	0.94	21.68	0.87	16.52	0.90
Moving Average (T=5)	-0.45	6.67	345.08	14.55	0.89	13.43	0.88	13.74	0.93
Moving Average (T=10)	-0.47	6.40	318.52	13.58	0.89	13.05	0.89	13.19	0.93
Simple Average	-0.31	6.17	298.82	12.96	0.88	12.88	0.90	12.91	0.90
Exponential Smoothing	-0.04	7.79	495.66	14.28	0.92	16.19	0.79	13.37	0.89
Regression	-0.26	6.18	299.44	12.84	0.88	12.84	0.91	12.84	0.89
AR(1)ULS	-0.28	6.09	290.82	11.95	0.88	14.39	0.95	12.67	0.90
AR(2)ULS	-0.32	5.48	228.73	8.98	0.92	18.03	1.00	15.46	0.97
MA(1)ULS	-0.29	6.04	285.26	11.66	0.86	12.83	0.90	12.83	0.90
MA(2)ULS	-0.57	8.14	828.54	21.21	0.73	12.83	0.90	12.83	0.90
ARMA(1,1)ULS	-0.30	6.18	299.10	12.84	0.87	12.83	0.90	12.83	0.90
ARMA(2,2) ULS	-0.40	5.68	250.55	9.57	0.87	12.97	0.96	12.88	0.91
ARMA(2,2) MLE	-0.31	5.51	232.37	8.69	0.88	12.92	0.97	12.81	0.92
Kalman Filter ARMA(2,2)	-0.28	6.15	296.72	12.17	0.89	13.00	0.90	12.89	0.90
MMAE ARMA(2,2)	-0.29	6.13	296.49	11.68	0.87	11.76	0.83	11.82	0.87

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	36.1	38.4

Table 56. Monte Carlo Simulation Results for Case 35

TRUE PARAMETERS

Phi1 =	0.6	Sample Size =	400
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	-0.6	Noise Std. Deviation =	5
Theta2=	-0.5	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.06	6.45	329.18	8.74	0.90	15.12	0.90	14.89	0.95
ARMA(2,2)InitialEstimates	-0.08	7.59	447.90	12.94	0.88	15.69	0.84	14.87	0.93
Naive	-0.41	9.42	740.41	12.77	0.90	24.03	0.93	20.77	0.92
Moving Average (T=5)	-0.02	8.00	511.50	16.79	0.91	17.34	0.91	16.99	0.91
Moving Average (T=10)	-0.34	7.46	448.21	15.67	0.91	16.15	0.91	15.94	0.93
Simple Average	-0.09	7.04	383.60	15.00	0.92	15.10	0.90	15.06	0.94
Exponential Smoothing	-0.38	9.05	683.13	14.35	0.93	21.40	0.92	18.42	0.89
Regression	-0.03	7.07	385.05	14.87	0.91	14.87	0.89	14.87	0.93
AR(1)ULS	-0.11	6.88	365.88	12.95	0.91	16.30	0.93	15.21	0.95
AR(2)ULS	0.06	7.19	425.00	9.63	0.91	20.84	0.95	19.77	0.97
MA(1)ULS	-0.10	6.88	366.11	12.92	0.92	14.88	0.89	14.88	0.92
MA(2)ULS	-4.11	20.87	580.86	66.34	0.58	14.88	0.89	14.88	0.92
ARMA(1,1)ULS	-0.63	8.33	551.83	13.43	0.92	19.30	0.88	17.65	0.93
ARMA(2,2) ULS	0.16	7.28	471.89	11.93	0.94	17.44	0.90	16.58	0.92
ARMA(2,2) MLE	-0.07	6.47	331.44	8.71	0.90	15.06	0.88	14.88	0.94
Kalman Filter ARMA(2,2)	-0.16	6.82	369.58	11.88	0.91	15.60	0.89	14.97	0.94
MMAE ARMA(2,2)	-0.19	6.85	372.65	10.39	0.87	11.69	0.83	12.13	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	45.9	45.1

Table 57. Monte Carlo Simulation Results for Case 36

TRUE PARAMETERS

Phi1 =	0.4	Sample Size =	300
Phi2 =	-0.3	Number of Predictions=	5
Theta1=	1	Noise Std. Deviation =	5
Theta2=	-0.6	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.07	4.48	159.87	8.64	0.89	9.97	0.91	9.95	0.91
ARMA(2,2)InitialEstimates	0.04	6.70	374.62	17.46	0.90	13.61	0.90	10.13	0.91
Naive	0.10	7.01	404.79	16.80	0.89	13.35	0.90	14.09	0.91
Moving Average (T=5)	-0.10	4.73	186.07	10.90	0.88	10.26	0.92	10.38	0.92
Moving Average (T=10)	-0.12	4.67	180.74	10.46	0.89	10.11	0.91	10.15	0.91
Simple Average	-0.04	4.57	172.47	10.08	0.89	9.98	0.90	9.99	0.92
Exponential Smoothing	0.02	5.59	262.69	12.06	0.85	10.11	0.87	10.68	0.88
Regression	0.00	4.58	173.01	9.97	0.88	9.97	0.89	9.97	0.92
AR(1)ULS	-0.04	4.49	164.73	9.20	0.94	10.44	0.90	10.21	0.92
AR(2)ULS	-0.05	4.51	162.29	8.60	0.89	11.70	0.96	10.99	0.93
MA(1)ULS	-0.04	4.50	164.76	9.14	0.92	9.95	0.90	9.95	0.92
MA(2)ULS	-0.37	4.78	286.40	9.88	0.89	9.95	0.90	9.95	0.92
ARMA(1,1)ULS	-0.04	4.57	172.31	9.93	0.89	9.95	0.90	9.95	0.92
ARMA(2,2) ULS	0.13	4.63	172.34	9.23	0.91	10.10	0.91	10.08	0.92
ARMA(2,2) MLE	-0.08	4.53	166.71	8.65	0.88	10.07	0.91	10.06	0.91
Kalman Filter ARMA(2,2)	-0.08	5.06	255.63	9.51	0.92	10.83	0.91	14.60	0.92
MMAE ARMA(2,2)	-0.05	5.12	258.86	9.15	0.92	9.43	0.87	9.51	0.85

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	38.9	35.7

Table 58. Monte Carlo Simulation Results for Case 37

TRUE PARAMETERS

Phi1 =	0.5	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.27	4.59	158.04	8.43	0.89	9.57	0.91	9.55	0.89
ARMA(2,2)InitialEstimates	-0.20	5.60	248.36	12.30	0.87	10.37	0.90	9.58	0.89
Naive	-0.13	5.97	289.88	9.83	0.88	12.81	0.90	13.38	0.91
Moving Average (T=5)	-0.49	5.27	209.59	9.93	0.83	11.27	0.92	11.38	0.94
Moving Average (T=10)	-0.47	5.17	202.29	9.88	0.85	10.60	0.91	10.66	0.91
Simple Average	-0.30	4.79	172.45	9.64	0.88	9.78	0.91	9.79	0.90
Exponential Smoothing	-0.25	5.40	229.28	9.16	0.89	11.22	0.90	11.64	0.90
Regression	-0.20	4.80	176.48	9.51	0.88	9.51	0.89	9.51	0.89
AR(1)ULS	-0.29	4.67	163.72	8.72	0.86	9.59	0.91	9.77	0.89
AR(2)ULS	-0.29	4.58	157.75	8.39	0.87	10.12	0.91	10.47	0.90
MA(1)ULS	-0.29	4.70	165.96	8.79	0.86	9.55	0.89	9.55	0.89
MA(2)ULS	0.23	13.57	16428.99	57.60	0.79	9.55	0.89	9.55	0.89
ARMA(1,1)ULS	-0.29	4.77	171.23	9.46	0.88	9.54	0.90	9.54	0.89
ARMA(2,2) ULS	-0.34	4.63	162.47	8.66	0.86	9.56	0.90	9.55	0.89
ARMA(2,2) MLE	-0.27	4.61	161.27	8.44	0.87	9.59	0.91	9.58	0.87
Kalman Filter ARMA(2,2)	-0.43	4.68	165.17	8.67	0.90	9.71	0.90	9.74	0.89
MMAE ARMA(2,2)	-0.35	4.66	163.45	8.50	0.89	9.30	0.89	9.56	0.89

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	45.3	38.1

Table 59. Monte Carlo Simulation Results for Case 38

TRUE PARAMETERS

Phi1 =	-0.5	Sample Size =	200
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	0.91	6.56	1.65	0.83	1.94	0.92	1.92	0.92
ARMA(2,2)InitialEstimates	-0.01	1.13	10.05	2.44	0.90	2.12	0.90	1.94	0.92
Naive	0.00	1.34	14.09	3.32	0.91	2.88	0.90	2.74	0.88
Moving Average (T=5)	-0.02	0.97	7.48	2.12	0.89	1.98	0.92	2.00	0.87
Moving Average (T=10)	0.00	0.95	7.22	2.02	0.88	1.95	0.93	1.95	0.88
Simple Average	0.00	0.94	6.95	1.95	0.88	1.92	0.92	1.93	0.92
Exponential Smoothing	-0.01	1.10	9.57	2.32	0.85	2.12	0.88	2.06	0.84
Regression	0.00	0.94	6.96	1.92	0.88	1.92	0.91	1.92	0.92
AR(1)ULS	0.00	0.92	6.72	1.73	0.87	1.91	0.93	1.96	0.93
AR(2)ULS	-0.01	0.92	6.63	1.64	0.83	2.02	0.93	2.12	0.94
MA(1)ULS	0.00	0.93	6.79	1.75	0.87	1.91	0.92	1.91	0.92
MA(2)ULS	-0.01	0.93	6.82	1.66	0.83	1.91	0.92	1.91	0.92
ARMA(1,1)ULS	0.00	0.94	6.97	1.93	0.88	1.91	0.92	1.92	0.92
ARMA(2,2) ULS	0.00	0.92	6.66	1.65	0.82	1.92	0.91	1.92	0.92
ARMA(2,2) MLE	0.00	0.93	6.76	1.67	0.83	1.94	0.91	1.92	0.92
Kalman Filter ARMA(2,2)	0.00	0.94	6.89	1.67	0.81	1.93	0.91	1.93	0.91
MMAE ARMA(2,2)	-0.01	0.94	7.01	1.87	0.87	2.11	0.94	2.14	0.92

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	54	52.4

Table 60. Monte Carlo Simulation Results for Case 39

TRUE PARAMETERS

Phi1 =	0.5	Sample Size =	200
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.89	6.17	1.65	0.84	1.89	0.90	1.88	0.95
ARMA(2,2)InitialEstimates	-0.03	1.11	10.09	2.41	0.90	2.04	0.88	1.88	0.94
Naive	-0.02	1.07	9.07	1.92	0.87	2.53	0.89	2.64	0.93
Moving Average (T=5)	-0.04	1.04	8.44	1.96	0.85	2.23	0.92	2.25	0.93
Moving Average (T=10)	0.01	1.01	7.77	1.95	0.85	2.09	0.91	2.11	0.91
Simple Average	-0.01	0.93	6.87	1.91	0.84	1.94	0.91	1.95	0.94
Exponential Smoothing	-0.03	1.02	8.15	1.80	0.86	2.23	0.87	2.32	0.90
Regression	-0.01	0.94	6.93	1.87	0.85	1.87	0.92	1.87	0.94
AR(1)ULS	-0.01	0.91	6.53	1.71	0.81	1.89	0.90	1.93	0.95
AR(2)ULS	-0.01	0.90	6.32	1.64	0.82	2.00	0.91	2.07	0.95
MA(1)ULS	-0.01	0.92	6.61	1.73	0.84	1.88	0.90	1.88	0.94
MA(2)ULS	-0.01	0.91	6.53	1.65	0.81	1.88	0.90	1.88	0.94
ARMA(1,1)ULS	0.01	0.90	6.44	1.82	0.84	2.14	0.90	2.12	0.97
ARMA(2,2) ULS	-0.01	0.91	6.41	1.65	0.81	1.88	0.90	1.88	0.94
ARMA(2,2) MLE	-0.01	0.91	6.40	1.66	0.82	1.89	0.88	1.89	0.94
Kalman Filter ARMA(2,2)	-0.01	0.90	6.35	1.66	0.84	1.89	0.87	1.90	0.94
MMAE ARMA(2,2)	-0.02	0.90	6.32	1.87	0.88	2.10	0.92	2.15	0.97

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	56.4	56

Table 61. Monte Carlo Simulation Results for Case 40

TRUE PARAMETERS

Phi1 =	-0.5	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.09	4.41	154.70	8.43	0.88	9.83	0.91	9.68	0.88
ARMA(2,2)InitialEstimates	-0.09	5.44	248.11	12.46	0.89	10.55	0.87	9.76	0.87
Naive	0.12	6.92	407.48	16.71	0.88	14.52	0.90	13.84	0.87
Moving Average (T=5)	-0.12	4.83	186.58	10.69	0.87	10.02	0.90	10.09	0.90
Moving Average (T=10)	-0.16	4.72	180.16	10.18	0.85	9.83	0.89	9.86	0.90
Simple Average	-0.08	4.65	171.53	9.80	0.86	9.70	0.90	9.71	0.89
Exponential Smoothing	0.02	5.61	263.86	11.67	0.86	10.71	0.86	10.42	0.89
Regression	-0.05	4.66	171.98	9.70	0.86	9.70	0.89	9.70	0.89
AR(1)ULS	-0.09	4.52	161.27	8.78	0.88	9.65	0.90	9.88	0.89
AR(2)ULS	-0.09	4.41	155.72	8.39	0.88	10.20	0.92	10.66	0.91
MA(1)ULS	-0.09	4.56	163.75	8.86	0.88	9.67	0.90	9.67	0.89
MA(2)ULS	-0.10	4.95	334.88	11.08	0.86	9.67	0.90	9.67	0.89
ARMA(1,1)ULS	-0.08	4.65	171.46	9.66	0.85	9.67	0.90	9.68	0.89
ARMA(2,2) ULS	0.02	4.72	243.95	9.65	0.86	9.90	0.90	9.73	0.87
ARMA(2,2) MLE	-0.09	4.46	158.83	8.47	0.88	10.01	0.93	10.00	0.89
Kalman Filter ARMA(2,2)	-0.01	5.35	271.68	8.71	0.90	10.66	0.90	15.57	0.88
MMAE ARMA(2,2)	-0.07	5.49	295.28	8.44	0.87	9.22	0.85	9.41	0.75

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	42.4	44

Table 62. Monte Carlo Simulation Results for Case 41

TRUE PARAMETERS

Phi1 =	0.8	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.12	5.54	246.09	8.20	0.92	12.40	0.90	13.21	0.88
ARMA(2,2)InitialEstimates	-0.18	7.26	404.26	11.99	0.88	14.28	0.85	15.12	0.87
Naive	-0.11	6.59	347.09	8.68	0.91	13.48	0.90	13.84	0.87
Moving Average (T=5)	-0.48	6.94	367.80	10.94	0.86	14.57	0.93	15.01	0.90
Moving Average (T=10)	-0.37	7.25	414.92	12.26	0.86	14.57	0.89	15.65	0.90
Simple Average	-0.14	6.86	367.33	13.39	0.88	13.77	0.88	14.21	0.89
Exponential Smoothing	-0.18	6.45	326.92	8.88	0.91	13.09	0.89	13.56	0.89
Regression	0.00	7.09	399.54	13.06	0.88	13.06	0.90	13.25	0.89
AR(1)ULS	-0.13	6.54	335.44	10.98	0.88	12.08	0.84	13.01	0.89
AR(2)ULS	-0.18	5.57	249.93	8.16	0.91	12.24	0.91	12.89	0.91
MA(1)ULS	-0.13	6.64	345.39	11.33	0.88	13.27	0.87	14.12	0.89
MA(2)ULS	2.29	53.78	164339.27	298.76	0.79	13.27	0.87	14.15	0.89
ARMA(1,1)ULS	-0.14	6.59	340.68	11.94	0.90	13.11	0.88	13.25	0.89
ARMA(2,2) ULS	-0.06	5.99	293.34	9.18	0.89	12.92	0.87	13.32	0.87
ARMA(2,2) MLE	-0.14	5.56	250.06	8.17	0.88	12.31	0.89	12.65	0.89
Kalman Filter ARMA(2,2)	-0.05	5.77	269.53	8.55	0.92	11.89	0.88	12.12	0.88
MMAE ARMA(2,2)	-0.12	5.62	256.61	8.17	0.91	10.90	0.83	11.23	0.86

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	50.5	47

Table 63. Monte Carlo Simulation Results for Case 42

TRUE PARAMETERS

Phi1 =	0.8	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.07	1.12	9.84	1.66	0.90	2.50	0.91	2.65	0.88
ARMA(2,2)InitialEstimates	-0.11	1.47	16.60	2.45	0.89	2.90	0.86	3.56	0.89
Naive	-0.05	1.31	13.50	1.76	0.88	2.71	0.90	3.12	0.91
Moving Average (T=5)	-0.08	1.41	14.98	2.20	0.83	2.93	0.92	3.25	0.90
Moving Average (T=10)	-0.06	1.46	16.67	2.47	0.84	2.93	0.89	3.06	0.90
Simple Average	-0.09	1.38	14.82	2.70	0.88	2.78	0.88	3.12	0.89
Exponential Smoothing	-0.06	1.28	12.82	1.79	0.89	2.64	0.88	2.98	0.86
Regression	-0.05	1.44	16.20	2.63	0.88	2.63	0.89	2.89	0.87
AR(1)ULS	-0.09	1.32	13.57	2.22	0.88	2.43	0.86	3.03	0.89
AR(2)ULS	-0.08	1.13	10.03	1.65	0.89	2.46	0.91	2.79	0.91
MA(1)ULS	-0.09	1.34	13.96	2.28	0.87	2.68	0.88	3.56	0.89
MA(2)ULS	-0.08	1.28	13.05	1.81	0.88	2.68	0.88	2.89	0.90
ARMA(1,1)ULS	-0.04	1.24	11.94	1.79	0.87	2.78	0.94	2.85	0.91
ARMA(2,2) ULS	-0.08	1.25	12.33	1.79	0.89	2.67	0.88	3.12	0.87
ARMA(2,2) MLE	-0.07	1.14	10.34	1.86	0.92	2.49	0.88	2.67	0.89
Kalman Filter ARMA(2,2)	-0.09	1.15	10.59	1.68	0.91	2.39	0.88	2.75	0.88
MMAE ARMA(2,2)	-0.08	1.12	10.04	1.91	0.94	2.56	0.91	2.68	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	41.6	50

Table 64. Monte Carlo Simulation Results for Case 43

TRUE PARAMETERS

Phi1 =	0.8	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.12	5.54	246.09	8.20	0.92	12.40	0.90	12.96	0.87
ARMA(2,2)InitialEstimates	-0.18	7.26	404.26	11.99	0.88	14.28	0.85	13.62	0.87
Naive	-0.11	6.59	347.09	8.68	0.91	13.48	0.90	15.68	0.87
Moving Average (T=5)	-0.48	6.94	367.80	10.94	0.86	14.57	0.93	15.69	0.93
Moving Average (T=10)	-0.37	7.25	414.92	12.26	0.86	14.57	0.89	15.33	0.88
Simple Average	-0.14	6.86	367.33	13.39	0.88	13.77	0.88	13.93	0.89
Exponential Smoothing	-0.18	6.45	326.92	8.88	0.91	13.09	0.89	15.09	0.88
Regression	0.00	7.09	399.54	13.06	0.88	13.06	0.90	13.06	0.85
AR(1)ULS	-0.13	6.54	335.44	10.98	0.88	12.08	0.84	12.69	0.86
AR(2)ULS	-0.18	5.57	249.93	8.16	0.91	12.24	0.91	14.16	0.93
MA(1)ULS	-0.13	6.64	345.39	11.33	0.88	13.27	0.87	13.27	0.88
MA(2)ULS	2.29	53.78	164339.27	298.76	0.79	13.27	0.87	13.27	0.88
ARMA(1,1)ULS	-0.14	6.59	340.68	11.94	0.90	13.11	0.88	13.29	0.87
ARMA(2,2) ULS	-0.06	5.99	293.34	9.18	0.89	12.92	0.87	13.20	0.88
ARMA(2,2) MLE	-0.14	5.56	250.06	8.17	0.88	12.31	0.89	12.89	0.87
Kalman Filter ARMA(2,2)	-0.05	5.77	269.53	8.55	0.92	11.89	0.88	12.94	0.86
MMAE ARMA(2,2)	-0.12	5.62	256.61	8.17	0.91	10.90	0.83	11.98	0.84

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	50.5	47

Table 65. Monte Carlo Simulation Results for Case 44

TRUE PARAMETERS

Phi1 =	0.8	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.07	1.12	9.84	1.66	0.90	2.50	0.91	2.61	0.90
ARMA(2,2)InitialEstimates	-0.11	1.47	16.60	2.45	0.89	2.90	0.86	2.76	0.88
Naive	-0.05	1.31	13.50	1.76	0.88	2.71	0.90	3.16	0.90
Moving Average (T=5)	-0.08	1.41	14.98	2.20	0.83	2.93	0.92	3.16	0.94
Moving Average (T=10)	-0.06	1.46	16.67	2.47	0.84	2.93	0.89	3.08	0.90
Simple Average	-0.09	1.38	14.82	2.70	0.88	2.78	0.88	2.81	0.88
Exponential Smoothing	-0.06	1.28	12.82	1.79	0.89	2.64	0.88	3.04	0.90
Regression	-0.05	1.44	16.20	2.63	0.88	2.63	0.89	2.63	0.88
AR(1)ULS	-0.09	1.32	13.57	2.22	0.88	2.43	0.86	2.56	0.86
AR(2)ULS	-0.08	1.13	10.03	1.65	0.89	2.46	0.91	2.85	0.93
MA(1)ULS	-0.09	1.34	13.96	2.28	0.87	2.68	0.88	2.67	0.87
MA(2)ULS	-0.08	1.28	13.05	1.81	0.88	2.68	0.88	2.67	0.87
ARMA(1,1)ULS	-0.04	1.24	11.94	1.79	0.87	2.78	0.94	3.02	0.92
ARMA(2,2) ULS	-0.08	1.25	12.33	1.79	0.89	2.67	0.88	2.67	0.87
ARMA(2,2) MLE	-0.07	1.14	10.34	1.86	0.92	2.49	0.88	2.62	0.89
Kalman Filter ARMA(2,2)	-0.09	1.15	10.59	1.68	0.91	2.39	0.88	2.59	0.87
MMAE ARMA(2,2)	-0.08	1.12	10.04	1.91	0.94	2.56	0.91	2.78	0.92

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	52	50

Table 66. Monte Carlo Simulation Results for Case 45

TRUE PARAMETERS

Phi1 =	-0.8	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.05	8.99	1.65	0.90	3.53	0.99	3.08	0.94
ARMA(2,2)InitialEstimates	0.00	1.39	15.38	2.50	0.88	2.71	0.84	2.73	0.90
Naive	0.04	1.90	33.80	5.19	0.88	4.75	0.87	4.43	0.90
Moving Average (T=5)	0.00	1.38	15.16	3.09	0.90	2.64	0.83	2.71	0.87
Moving Average (T=10)	-0.01	1.35	14.55	2.87	0.88	2.70	0.87	2.72	0.89
Simple Average	0.00	1.33	14.09	2.78	0.89	2.72	0.87	2.73	0.91
Exponential Smoothing	0.02	1.55	21.49	3.40	0.82	3.21	0.82	3.08	0.82
Regression	0.01	1.33	14.12	2.75	0.89	2.75	0.90	2.75	0.89
AR(1)ULS	0.00	1.27	12.70	2.26	0.87	2.47	0.85	2.61	0.87
AR(2)ULS	0.00	1.05	9.03	1.65	0.91	2.48	0.92	2.91	0.93
MA(1)ULS	0.00	1.29	13.14	2.33	0.88	2.74	0.88	2.73	0.90
MA(2)ULS	-0.01	1.23	12.06	1.82	0.87	2.74	0.88	2.73	0.90
ARMA(1,1)ULS	0.00	1.32	13.80	2.66	0.88	2.74	0.90	2.74	0.90
ARMA(2,2) ULS	0.00	1.05	9.03	1.65	0.90	2.88	0.96	2.76	0.93
ARMA(2,2) MLE	0.00	1.09	9.48	1.67	0.89	3.20	0.94	3.07	0.92
Kalman Filter ARMA(2,2)	-0.01	1.42	22.83	1.70	0.90	3.04	0.90	4.69	0.89
MMAE ARMA(2,2)	-0.02	1.46	26.62	1.91	0.93	2.57	0.84	2.76	0.80

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	41	39.5

Table 67. Monte Carlo Simulation Results for Case 46

TRUE PARAMETERS

Phi1 =	-0.2	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.19	3.98	126.34	8.43	0.86	8.61	0.94	8.61	0.91
ARMA(2,2)InitialEstimates	-0.13	4.80	195.99	12.37	0.91	8.95	0.94	8.61	0.91
Naive	-0.06	5.86	288.51	13.36	0.89	12.28	0.92	12.15	0.90
Moving Average (T=5)	-0.28	4.24	142.65	9.49	0.90	9.21	0.96	9.22	0.92
Moving Average (T=10)	-0.27	4.12	136.70	9.06	0.88	8.91	0.94	8.91	0.91
Simple Average	-0.18	4.00	128.05	8.71	0.89	8.67	0.94	8.68	0.91
Exponential Smoothing	-0.17	4.85	193.31	10.03	0.85	9.49	0.88	9.45	0.87
Regression	-0.13	4.00	128.19	8.62	0.89	8.62	0.94	8.62	0.91
AR(1)ULS	-0.19	3.98	126.32	8.43	0.86	8.81	0.94	8.85	0.91
AR(2)ULS	-0.18	3.98	126.50	8.39	0.86	8.76	0.93	8.80	0.91
MA(1)ULS	-0.19	3.99	126.86	8.43	0.86	8.61	0.94	8.61	0.91
MA(2)ULS	-0.18	3.98	126.18	8.40	0.87	8.61	0.94	8.61	0.91
ARMA(1,1)ULS	-0.18	4.00	128.03	8.61	0.89	8.61	0.94	8.61	0.91
ARMA(2,2) ULS	-0.26	4.10	136.59	8.89	0.88	8.64	0.94	8.62	0.91
ARMA(2,2) MLE	-0.20	4.06	133.50	8.56	0.87	8.76	0.94	8.79	0.91
Kalman Filter ARMA(2,2)	-0.25	4.20	143.31	8.53	0.89	8.99	0.92	10.58	0.89
MMAE ARMA(2,2)	-0.28	4.28	152.86	8.44	0.89	8.55	0.91	8.59	0.85

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	49	50

Table 68. Monte Carlo Simulation Results for Case 47

TRUE PARAMETERS

Phi1 =	0.9	Sample Size =	300
Phi2 =	-0.6	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.34	5.40	226.52	8.58	0.89	12.85	0.96	12.46	0.88
ARMA(2,2)InitialEstimates	-0.17	7.91	523.17	15.66	0.85	13.78	0.85	12.52	0.87
Naive	-0.13	8.96	673.57	11.90	0.90	21.12	0.89	18.07	0.88
Moving Average (T=5)	-0.56	6.98	373.34	14.79	0.92	14.01	0.91	13.45	0.92
Moving Average (T=10)	-0.60	6.59	332.42	13.25	0.85	13.30	0.92	12.83	0.89
Simple Average	-0.40	6.28	301.30	12.65	0.87	12.69	0.94	12.60	0.89
Exponential Smoothing	-0.24	8.45	592.03	12.95	0.92	18.65	0.87	15.38	0.85
Regression	-0.35	6.26	302.72	12.53	0.87	12.53	0.94	12.53	0.89
AR(1)ULS	-0.38	6.11	284.77	11.17	0.87	14.03	0.95	12.99	0.90
AR(2)ULS	-0.35	5.40	225.52	8.53	0.88	17.16	1.00	16.69	0.99
MA(1)ULS	-0.39	6.10	283.78	11.07	0.88	12.52	0.93	12.52	0.89
MA(2)ULS	-2.31	23.17	17785.69	97.62	0.62	12.52	0.93	12.52	0.89
ARMA(1,1)ULS	-0.39	6.27	300.97	12.50	0.88	12.53	0.93	12.52	0.89
ARMA(2,2) ULS	-3.64	9.10	3951.29	11.69	0.85	15.37	0.93	12.68	0.88
ARMA(2,2) MLE	-0.39	5.43	229.20	8.52	0.87	12.83	0.95	12.46	0.90
Kalman Filter ARMA(2,2)	-0.32	6.13	289.29	11.36	0.88	12.77	0.94	12.61	0.89
MMAE ARMA(2,2)	-0.34	6.15	290.29	10.78	0.86	11.08	0.81	11.22	0.84

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	33.5	34.5

Table 69. Monte Carlo Simulation Results for Case 48

TRUE PARAMETERS

Phi1 =	-0.9	Sample Size =	300
Phi2 =	-0.6	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.02	5.20	214.46	8.57	0.89	14.55	0.95	13.42	0.91
ARMA(2,2)InitialEstimates	0.02	6.68	361.63	13.16	0.88	24.45	0.99	17.56	0.95
Naive	0.01	9.11	676.97	22.15	0.85	13.68	0.92	17.36	0.90
Moving Average (T=5)	-0.09	6.26	310.36	14.01	0.89	13.38	0.91	12.80	0.90
Moving Average (T=10)	-0.08	6.16	300.46	13.27	0.88	12.97	0.91	12.60	0.90
Simple Average	-0.01	6.08	291.35	12.73	0.89	12.65	0.90	12.57	0.87
Exponential Smoothing	-0.02	7.37	439.83	15.51	0.81	11.90	0.93	12.74	0.86
Regression	0.01	6.07	291.89	12.60	0.89	12.60	0.90	12.60	0.90
AR(1)ULS	-0.02	5.93	275.71	11.19	0.89	14.03	0.94	13.03	0.91
AR(2)ULS	-0.02	5.24	216.46	8.53	0.89	17.19	0.99	16.74	0.98
MA(1)ULS	-0.02	5.92	274.72	11.10	0.89	12.56	0.89	12.56	0.87
MA(2)ULS	0.78	9.23	3283.73	24.22	0.78	12.56	0.89	12.56	0.87
ARMA(1,1)ULS	-0.01	6.08	291.35	12.54	0.89	12.56	0.89	12.57	0.87
ARMA(2,2) ULS	-0.15	5.66	267.08	10.21	0.91	17.70	0.95	22.74	0.92
ARMA(2,2) MLE	-0.02	5.21	214.05	8.52	0.92	14.44	0.95	13.51	0.91
Kalman Filter ARMA(2,2)	-0.15	8.00	1191.12	11.25	0.91	15.09	0.91	23.10	0.91
MMAE ARMA(2,2)	-0.18	8.38	1325.10	10.57	0.89	10.93	0.77	11.06	0.72

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	27.3	22.6

Table 70. Monte Carlo Simulation Results for Case 49

TRUE PARAMETERS

Phi1 =	0.5	Sample Size =	300
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.34	4.82	175.99	8.44	0.90	10.34	0.91	10.56	0.89
ARMA(2,2)InitialEstimates	-0.51	6.35	319.98	12.92	0.87	12.38	0.88	11.16	0.88
Naive	-0.29	5.87	277.57	9.65	0.86	12.21	0.88	13.53	0.90
Moving Average (T=5)	-0.60	5.58	236.73	9.73	0.87	11.97	0.91	12.63	0.93
Moving Average (T=10)	-0.53	5.71	254.13	10.35	0.85	11.75	0.88	12.17	0.91
Simple Average	-0.33	5.37	224.16	10.76	0.89	11.00	0.92	11.08	0.86
Exponential Smoothing	-0.44	5.54	239.98	8.85	0.84	11.23	0.89	12.42	0.89
Regression	-0.21	5.51	235.36	10.56	0.87	10.56	0.91	10.56	0.88
AR(1)ULS	-0.32	5.20	210.00	9.40	0.84	10.08	0.88	10.47	0.86
AR(2)ULS	-0.36	4.83	178.22	8.40	0.88	10.33	0.91	11.33	0.90
MA(1)ULS	-0.32	5.25	215.12	9.61	0.85	10.66	0.91	10.66	0.86
MA(2)ULS	5.94	33.32	53702.48	184.59	0.76	10.66	0.91	10.66	0.86
ARMA(1,1)ULS	-0.31	5.31	218.61	10.33	0.89	10.62	0.92	10.65	0.86
ARMA(2,2) ULS	-0.25	4.91	186.94	9.01	0.89	10.43	0.94	10.60	0.87
ARMA(2,2) MLE	-0.35	4.85	179.20	8.40	0.88	10.27	0.91	10.53	0.89
Kalman Filter ARMA(2,2)	-0.46	4.95	187.90	8.70	0.89	10.23	0.92	10.69	0.87
MMAE ARMA(2,2)	-0.35	4.84	179.58	8.55	0.90	9.80	0.89	10.29	0.88

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	52.4	48.4

Table 71. Monte Carlo Simulation Results for Case 50

TRUE PARAMETERS

Phi1 =	-0.5	Sample Size =	300
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.10	4.57	168.42	8.43	0.87	11.49	0.94	10.99	0.91
ARMA(2,2)InitialEstimates	-0.08	5.81	276.29	12.54	0.90	10.78	0.85	10.86	0.88
Naive	0.09	7.58	519.87	19.56	0.88	18.03	0.84	16.89	0.89
Moving Average (T=5)	-0.21	5.50	239.33	12.18	0.88	10.95	0.83	11.14	0.89
Moving Average (T=10)	-0.21	5.39	231.12	11.43	0.87	10.94	0.85	11.01	0.88
Simple Average	-0.09	5.28	219.29	11.04	0.89	10.89	0.86	10.91	0.91
Exponential Smoothing	-0.04	6.25	334.44	13.21	0.84	12.63	0.81	12.18	0.80
Regression	-0.06	5.29	220.05	10.92	0.88	10.92	0.87	10.92	0.90
AR(1)ULS	-0.10	5.09	202.26	9.53	0.86	10.21	0.86	10.65	0.88
AR(2)ULS	-0.11	4.56	168.57	8.39	0.88	10.43	0.90	11.59	0.91
MA(1)ULS	-0.09	5.16	208.13	9.76	0.85	10.89	0.86	10.88	0.90
MA(2)ULS	-3.99	15.33	50213.16	55.23	0.70	10.89	0.86	10.88	0.90
ARMA(1,1)ULS	-0.09	5.28	218.95	10.87	0.88	10.89	0.86	10.89	0.90
ARMA(2,2) ULS	-0.07	4.65	176.65	8.48	0.87	11.36	0.92	10.98	0.91
ARMA(2,2) MLE	-0.11	4.58	170.12	8.42	0.87	11.94	0.94	11.40	0.91
Kalman Filter ARMA(2,2)	-0.05	6.12	467.12	9.00	0.87	11.61	0.90	18.06	0.89
MMAE ARMA(2,2)	-0.05	6.32	502.18	8.80	0.86	9.97	0.86	10.35	0.68

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	31.2	34.4

Table 72. Monte Carlo Simulation Results for Case 51

TRUE PARAMETERS

Phi1 =	-0.5	Sample Size =	200
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.01	4.80	178.03	8.46	0.85	10.66	0.96	10.23	0.93
ARMA(2,2)InitialEstimates	0.03	5.78	265.18	13.47	0.92	15.02	0.97	10.26	0.91
Naive	-0.26	7.21	410.95	16.66	0.90	11.98	0.93	15.54	0.89
Moving Average (T=5)	-0.14	5.17	210.71	11.50	0.88	10.77	0.94	10.42	0.86
Moving Average (T=10)	-0.05	5.05	201.82	10.77	0.88	10.45	0.92	10.29	0.87
Simple Average	-0.02	4.98	193.68	10.42	0.88	10.29	0.89	10.25	0.90
Exponential Smoothing	-0.17	5.95	279.62	12.43	0.86	9.74	0.94	11.30	0.80
Regression	-0.03	4.98	194.41	10.27	0.89	10.27	0.89	10.27	0.91
AR(1)ULS	-0.02	4.97	192.80	9.71	0.88	11.21	0.94	10.13	0.90
AR(2)ULS	0.00	4.83	180.03	8.39	0.82	13.05	0.99	11.38	0.94
MA(1)ULS	-0.01	4.93	190.06	9.52	0.88	10.22	0.89	10.23	0.90
MA(2)ULS	-0.02	4.93	191.13	8.89	0.85	10.22	0.89	10.23	0.90
ARMA(1,1)ULS	-0.02	4.98	193.66	10.23	0.88	10.23	0.89	10.23	0.90
ARMA(2,2) ULS	0.02	4.87	182.18	8.46	0.83	10.59	0.95	10.23	0.93
ARMA(2,2) MLE	0.01	4.85	182.60	8.44	0.83	10.63	0.94	10.27	0.91
Kalman Filter ARMA(2,2)	0.00	5.07	195.49	9.89	0.87	10.48	0.91	11.49	0.88
MMAE ARMA(2,2)	0.02	5.10	199.23	9.39	0.85	9.45	0.89	9.49	0.86

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	48	49.6

Table 73. Monte Carlo Simulation Results for Case 52

TRUE PARAMETERS

Phi1 =	0.5	Sample Size =	200
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.07	4.75	177.64	8.46	0.83	10.28	0.88	10.25	0.89
ARMA(2,2)InitialEstimates	0.00	6.22	326.82	13.84	0.88	10.39	0.85	10.26	0.88
Naive	0.02	6.93	376.28	11.96	0.85	16.75	0.91	13.22	0.90
Moving Average (T=5)	-0.17	5.39	228.66	11.44	0.85	10.82	0.87	11.09	0.89
Moving Average (T=10)	-0.24	5.19	213.57	10.82	0.83	10.47	0.88	10.63	0.89
Simple Average	-0.06	5.05	199.40	10.40	0.85	10.32	0.90	10.37	0.92
Exponential Smoothing	-0.03	5.96	279.34	11.44	0.87	12.52	0.87	10.83	0.91
Regression	-0.04	5.08	199.86	10.26	0.84	10.26	0.89	10.26	0.92
AR(1)ULS	-0.06	4.99	196.39	9.74	0.84	11.29	0.91	10.13	0.91
AR(2)ULS	-0.07	4.81	183.35	8.39	0.82	13.12	0.94	11.43	0.93
MA(1)ULS	-0.06	4.96	193.46	9.55	0.84	10.24	0.90	10.25	0.92
MA(2)ULS	-0.08	4.95	190.73	9.11	0.86	10.24	0.90	10.25	0.92
ARMA(1,1)ULS	-0.06	5.05	199.46	10.27	0.84	10.24	0.90	10.25	0.91
ARMA(2,2) ULS	0.00	4.86	185.65	8.38	0.80	10.26	0.88	10.24	0.91
ARMA(2,2) MLE	-0.04	4.90	188.63	8.49	0.82	10.34	0.89	10.27	0.91
Kalman Filter ARMA(2,2)	-0.07	5.06	199.18	9.80	0.82	10.41	0.90	10.32	0.92
MMAE ARMA(2,2)	-0.08	5.05	198.32	9.23	0.81	9.31	0.84	9.39	0.88

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	44.4	44

Table 74. Monte Carlo Simulation Results for Case 53

TRUE PARAMETERS

Phi1 =	0.5	Sample Size =	300
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.05	0.96	7.04	1.66	0.89	2.04	0.90	2.09	0.88
ARMA(2,2)InitialEstimates	-0.05	1.25	12.25	2.60	0.89	2.46	0.89	2.21	0.87
Naive	-0.06	1.17	10.95	1.89	0.84	2.41	0.88	2.67	0.89
Moving Average (T=5)	-0.10	1.12	9.57	1.92	0.86	2.37	0.90	2.51	0.90
Moving Average (T=10)	-0.08	1.15	10.20	2.05	0.85	2.33	0.88	2.42	0.91
Simple Average	-0.03	1.07	8.89	2.13	0.88	2.18	0.90	2.19	0.88
Exponential Smoothing	-0.08	1.11	9.62	1.74	0.86	2.22	0.90	2.47	0.88
Regression	-0.01	1.10	9.36	2.09	0.88	2.09	0.90	2.09	0.87
AR(1)ULS	-0.03	1.04	8.33	1.86	0.86	1.99	0.88	2.07	0.86
AR(2)ULS	-0.05	0.96	7.10	1.65	0.88	2.04	0.90	2.25	0.91
MA(1)ULS	-0.03	1.05	8.53	1.90	0.84	2.11	0.89	2.11	0.86
MA(2)ULS	-0.03	1.02	8.17	1.72	0.90	2.11	0.89	2.11	0.86
ARMA(1,1)ULS	-0.05	1.05	8.23	1.71	0.86	2.23	0.88	2.33	0.90
ARMA(2,2) ULS	-0.03	1.01	7.85	1.71	0.90	2.11	0.90	2.11	0.86
ARMA(2,2) MLE	-0.04	0.97	7.21	1.69	0.90	2.04	0.90	2.09	0.88
Kalman Filter ARMA(2,2)	-0.07	0.97	7.24	1.66	0.89	1.99	0.89	2.08	0.87
MMAE ARMA(2,2)	-0.06	0.97	7.19	1.87	0.92	2.20	0.93	2.32	0.93

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	58.8	56.4

Table 75. Monte Carlo Simulation Results for Case 54

TRUE PARAMETERS

Phi1 =	-0.5	Sample Size =	300
Phi2 =	0.2	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.02	0.90	6.61	1.65	0.88	2.28	0.94	2.18	0.91
ARMA(2,2)InitialEstimates	-0.02	1.15	10.80	2.49	0.90	2.14	0.83	2.15	0.88
Naive	0.02	1.51	20.57	3.89	0.88	3.58	0.84	3.35	0.90
Moving Average (T=5)	-0.03	1.09	9.40	2.42	0.88	2.17	0.84	2.21	0.90
Moving Average (T=10)	-0.04	1.07	9.08	2.27	0.86	2.17	0.84	2.18	0.88
Simple Average	-0.02	1.05	8.62	2.19	0.87	2.16	0.87	2.16	0.90
Exponential Smoothing	0.00	1.24	13.20	2.62	0.84	2.51	0.82	2.41	0.80
Regression	-0.01	1.05	8.65	2.16	0.88	2.16	0.88	2.16	0.89
AR(1)ULS	-0.02	1.01	7.94	1.88	0.86	2.02	0.86	2.11	0.89
AR(2)ULS	-0.02	0.90	6.61	1.65	0.88	2.07	0.91	2.30	0.91
MA(1)ULS	-0.02	1.02	8.17	1.93	0.85	2.16	0.87	2.16	0.90
MA(2)ULS	-0.02	0.99	7.75	1.74	0.86	2.16	0.87	2.16	0.90
ARMA(1,1)ULS	-0.02	1.04	8.57	2.16	0.87	2.16	0.87	2.16	0.90
ARMA(2,2) ULS	-0.02	0.90	6.60	1.65	0.88	2.21	0.93	2.17	0.90
ARMA(2,2) MLE	-0.02	0.92	6.78	1.67	0.88	2.30	0.91	2.24	0.91
Kalman Filter ARMA(2,2)	-0.01	1.06	11.34	1.68	0.88	2.22	0.89	2.61	0.90
MMAE ARMA(2,2)	-0.01	1.07	13.02	1.87	0.92	2.22	0.88	2.33	0.85

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	35.2	33.6

Table 76. Monte Carlo Simulation Results for Case 55

TRUE PARAMETERS

Phi1 =	-0.5	Sample Size =	200
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	0.94	6.94	1.66	0.84	2.11	0.96	2.03	0.90
ARMA(2,2)InitialEstimates	0.00	1.16	10.73	2.74	0.91	3.04	0.95	2.03	0.88
Naive	-0.05	1.42	16.04	3.31	0.90	2.36	0.92	3.08	0.88
Moving Average (T=5)	-0.03	1.02	8.27	2.28	0.89	2.13	0.93	2.06	0.82
Moving Average (T=10)	-0.01	1.00	7.91	2.13	0.89	2.07	0.91	2.04	0.88
Simple Average	-0.01	0.99	7.60	2.06	0.91	2.04	0.89	2.03	0.88
Exponential Smoothing	-0.04	1.18	10.95	2.47	0.86	1.92	0.93	2.24	0.78
Regression	-0.01	0.98	7.63	2.03	0.90	2.03	0.89	2.03	0.88
AR(1)ULS	-0.01	0.98	7.55	1.92	0.88	2.22	0.94	2.01	0.88
AR(2)ULS	0.00	0.95	7.05	1.64	0.82	2.61	0.99	2.26	0.94
MA(1)ULS	-0.01	0.97	7.44	1.88	0.89	2.02	0.89	2.03	0.87
MA(2)ULS	-0.01	0.97	7.47	1.75	0.85	2.02	0.89	2.03	0.87
ARMA(1,1)ULS	-0.01	0.99	7.61	2.04	0.90	2.03	0.89	2.03	0.87
ARMA(2,2) ULS	0.00	0.96	7.18	1.69	0.84	2.03	0.92	2.03	0.88
ARMA(2,2) MLE	0.00	0.95	7.11	1.66	0.82	2.08	0.94	2.03	0.89
Kalman Filter ARMA(2,2)	0.00	0.99	7.73	1.67	0.83	2.08	0.93	2.23	0.88
MMAE ARMA(2,2)	0.00	1.00	7.74	1.89	0.87	2.14	0.93	2.23	0.91

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	38.4	41.2

Table 77. Monte Carlo Simulation Results for Case 56

TRUE PARAMETERS

Phi1 =	0.5	Sample Size =	200
Phi2 =	-0.5	Number of Predictions=	5
Theta1=	0	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.93	6.82	1.66	0.83	2.03	0.89	2.03	0.91
ARMA(2,2)InitialEstimates	0.00	1.21	12.29	2.75	0.93	2.05	0.85	2.03	0.86
Naive	0.02	1.37	14.69	2.35	0.86	3.32	0.92	2.61	0.90
Moving Average (T=5)	-0.03	1.06	8.86	2.26	0.84	2.14	0.87	2.19	0.91
Moving Average (T=10)	-0.04	1.03	8.26	2.14	0.85	2.07	0.87	2.10	0.91
Simple Average	-0.01	0.99	7.71	2.06	0.86	2.04	0.89	2.05	0.92
Exponential Smoothing	0.01	1.18	10.85	2.26	0.88	2.48	0.88	2.14	0.90
Regression	-0.01	1.00	7.72	2.03	0.86	2.03	0.88	2.03	0.91
AR(1)ULS	-0.01	0.98	7.59	1.92	0.84	2.23	0.91	2.00	0.91
AR(2)ULS	-0.01	0.95	7.03	1.64	0.81	2.62	0.94	2.27	0.94
MA(1)ULS	-0.01	0.98	7.48	1.88	0.85	2.03	0.89	2.03	0.92
MA(2)ULS	-0.01	0.97	7.38	1.76	0.84	2.03	0.89	2.03	0.92
ARMA(1,1)ULS	0.00	1.00	7.80	2.03	0.85	2.03	0.87	2.03	0.92
ARMA(2,2) ULS	-0.01	0.94	7.07	1.67	0.82	2.04	0.88	2.03	0.92
ARMA(2,2) MLE	0.00	0.95	7.07	1.66	0.83	2.04	0.88	2.03	0.91
Kalman Filter ARMA(2,2)	-0.01	0.97	7.26	1.67	0.83	2.06	0.89	2.04	0.92
MMAE ARMA(2,2)	-0.01	0.97	7.31	1.89	0.86	2.13	0.91	2.23	0.96

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	41.6	44.8

Table 78. Monte Carlo Simulation Results for Case 57

TRUE PARAMETERS

Phi1 =	0	Sample Size =	500
Phi2 =	0	Number of Predictions=	5
Theta1=	-0.9	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.05	5.11	207.66	9.11	0.89	11.15	0.92	11.13	0.88
ARMA(2,2)InitialEstimates	-0.10	5.48	240.36	9.71	0.88	11.31	0.92	11.14	0.88
Naive	0.10	7.48	441.13	11.34	0.91	15.78	0.88	15.77	0.90
Moving Average (T=5)	0.03	6.40	323.97	12.02	0.92	12.97	0.89	12.96	0.90
Moving Average (T=10)	-0.05	6.03	284.79	11.64	0.92	12.15	0.89	12.15	0.90
Simple Average	-0.11	5.38	228.30	11.21	0.92	11.27	0.92	11.27	0.88
Exponential Smoothing	0.06	6.86	373.51	11.20	0.88	13.48	0.86	13.52	0.91
Regression	-0.19	5.41	230.79	11.13	0.89	11.13	0.92	11.13	0.89
AR(1)ULS	-0.10	5.27	219.25	10.13	0.90	11.45	0.92	11.45	0.88
AR(2)ULS	0.10	5.54	246.88	10.16	0.90	15.01	0.96	14.98	0.98
MA(1)ULS	-0.10	5.26	218.45	10.07	0.91	11.13	0.92	11.13	0.88
MA(2)ULS	-5.91	36.24	253382.55	98.01	0.66	11.13	0.92	11.13	0.88
ARMA(1,1)ULS	-0.11	5.37	228.13	11.13	0.92	11.13	0.92	11.13	0.88
ARMA(2,2) ULS	-0.28	5.45	234.77	10.14	0.90	11.96	0.93	11.56	0.91
ARMA(2,2) MLE	0.04	5.18	214.94	8.94	0.88	11.20	0.93	11.17	0.88
Kalman Filter ARMA(2,2)	-0.01	5.40	231.31	10.23	0.87	11.27	0.92	11.20	0.89
MMAE ARMA(2,2)	-0.06	5.42	232.79	9.81	0.86	10.63	0.89	10.85	0.86

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	38.7	34.8

Table 79. Monte Carlo Simulation Results for Case 58

TRUE PARAMETERS

Phi1 =	0	Sample Size =	500
Phi2 =	0	Number of Predictions=	5
Theta1=	-0.9	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.01	8.08	1.70	0.90	2.21	0.91	2.21	0.90
ARMA(2,2)InitialEstimates	-0.02	1.07	9.16	1.84	0.89	2.24	0.92	2.21	0.89
Naive	0.04	1.49	17.41	2.23	0.90	3.13	0.87	3.13	0.90
Moving Average (T=5)	0.01	1.27	12.68	2.38	0.92	2.57	0.88	2.57	0.90
Moving Average (T=10)	0.00	1.19	11.21	2.31	0.91	2.41	0.87	2.41	0.90
Simple Average	-0.02	1.07	9.00	2.22	0.90	2.24	0.92	2.24	0.90
Exponential Smoothing	0.03	1.37	14.84	2.22	0.88	2.69	0.86	2.70	0.92
Regression	-0.03	1.08	9.11	2.21	0.90	2.21	0.91	2.21	0.90
AR(1)ULS	-0.01	1.04	8.63	2.01	0.88	2.27	0.93	2.27	0.90
AR(2)ULS	-0.01	1.03	8.31	1.82	0.91	2.54	0.97	2.53	0.94
MA(1)ULS	-0.01	1.04	8.60	1.99	0.89	2.21	0.91	2.21	0.90
MA(2)ULS	-0.01	1.01	8.13	1.68	0.88	2.21	0.91	2.21	0.90
ARMA(1,1)ULS	-0.02	1.06	8.94	2.17	0.90	2.21	0.90	2.21	0.90
ARMA(2,2) ULS	-0.01	1.01	8.14	1.69	0.88	2.21	0.92	2.21	0.90
ARMA(2,2) MLE	0.00	1.01	8.13	1.69	0.89	2.21	0.93	2.21	0.90
Kalman Filter ARMA(2,2)	0.00	1.01	8.06	1.70	0.90	2.22	0.90	2.22	0.90
MMAE ARMA(2,2)	-0.01	1.02	8.25	1.92	0.92	2.36	0.92	2.41	0.94

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	46.8	42

Table 80. Monte Carlo Simulation Results for Case 59

TRUE PARAMETERS

Phi1 =	0	Sample Size =	500
Phi2 =	0	Number of Predictions=	5
Theta1=	0.9	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.02	0.99	7.55	1.71	0.90	2.23	0.93	2.23	0.90
ARMA(2,2)InitialEstimates	0.03	1.07	8.75	2.15	0.90	2.45	0.89	2.25	0.92
Naive	0.09	1.50	18.90	3.85	0.90	3.16	0.90	3.14	0.91
Moving Average (T=5)	0.04	1.09	9.14	2.48	0.87	2.27	0.91	2.28	0.89
Moving Average (T=10)	0.03	1.07	8.57	2.35	0.91	2.24	0.92	2.24	0.89
Simple Average	0.03	1.04	8.31	2.25	0.89	2.23	0.93	2.23	0.91
Exponential Smoothing	0.07	1.25	12.60	2.74	0.85	2.34	0.86	2.34	0.87
Regression	0.03	1.04	8.32	2.23	0.89	2.23	0.93	2.23	0.90
AR(1)ULS	0.03	1.02	7.99	2.02	0.89	2.28	0.93	2.29	0.92
AR(2)ULS	0.02	1.03	8.08	1.82	0.90	2.56	0.94	2.56	0.94
MA(1)ULS	0.03	1.02	7.98	2.01	0.90	2.23	0.93	2.23	0.90
MA(2)ULS	0.02	0.99	7.58	1.68	0.90	2.23	0.93	2.23	0.90
ARMA(1,1)ULS	0.03	1.04	8.30	2.21	0.90	2.23	0.93	2.23	0.90
ARMA(2,2) ULS	0.02	0.99	7.58	1.70	0.90	2.23	0.94	2.23	0.90
ARMA(2,2) MLE	0.02	1.00	7.67	1.72	0.89	2.24	0.93	2.23	0.90
Kalman Filter ARMA(2,2)	0.02	0.99	7.57	1.69	0.90	2.26	0.93	2.25	0.93
MMAE ARMA(2,2)	0.02	0.99	7.59	1.91	0.94	2.33	0.94	2.35	0.93

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	50.8	52

Table 81. Monte Carlo Simulation Results for Case 60

TRUE PARAMETERS

Phi1 =	0	Sample Size =	200
Phi2 =	0	Number of Predictions=	5
Theta1=	-0.5	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.18	4.49	157.85	8.46	0.84	9.33	0.89	9.53	0.87
ARMA(2,2)InitialEstimates	-0.15	5.30	233.45	11.34	0.86	9.58	0.87	9.75	0.86
Naive	-0.10	5.75	259.06	10.42	0.85	13.32	0.89	13.12	0.88
Moving Average (T=5)	-0.39	5.10	203.64	10.16	0.84	10.74	0.90	10.95	0.89
Moving Average (T=10)	-0.19	4.90	186.97	9.78	0.83	10.09	0.90	10.56	0.90
Simple Average	-0.19	4.61	168.56	9.46	0.83	9.55	0.91	10.23	0.91
Exponential Smoothing	-0.26	5.19	211.85	9.68	0.84	11.04	0.86	11.25	0.87
Regression	-0.13	4.64	169.37	9.32	0.81	9.32	0.91	9.65	0.90
AR(1)ULS	-0.18	4.53	161.84	8.74	0.83	9.63	0.89	9.70	0.85
AR(2)ULS	-0.16	4.55	163.77	8.42	0.82	10.15	0.91	10.89	0.86
MA(1)ULS	-0.18	4.53	162.02	8.69	0.83	9.32	0.89	9.56	0.90
MA(2)ULS	-0.19	4.52	159.86	8.42	0.83	9.32	0.89	9.56	0.90
ARMA(1,1)ULS	-0.18	4.61	168.35	9.32	0.83	9.33	0.89	9.64	0.90
ARMA(2,2) ULS	-0.15	4.56	164.24	8.39	0.83	9.34	0.89	9.86	0.91
ARMA(2,2) MLE	-0.17	4.58	166.15	8.45	0.85	9.40	0.89	9.52	0.90
Kalman Filter ARMA(2,2)	-0.03	4.56	164.56	9.01	0.81	9.46	0.90	10.05	0.89
MMAE ARMA(2,2)	-0.05	4.57	164.21	8.82	0.81	9.19	0.87	9.35	0.89

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	49	49.5

Table 82. Monte Carlo Simulation Results for Case 61

TRUE PARAMETERS

Phi1 =	0	Sample Size =	200
Phi2 =	0	Number of Predictions=	5
Theta1=	-0.5	Noise Std. Deviation =	5
Theta2=	0	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.18	4.49	157.85	8.46	0.84	9.33	0.89	9.32	0.94
ARMA(2,2)InitialEstimates	-0.15	5.30	233.45	11.34	0.86	9.58	0.87	9.32	0.94
Naive	-0.10	5.75	259.06	10.42	0.85	13.32	0.89	13.22	0.91
Moving Average (T=5)	-0.39	5.10	203.64	10.16	0.84	10.74	0.90	10.72	0.89
Moving Average (T=10)	-0.19	4.90	186.97	9.78	0.83	10.09	0.90	10.08	0.92
Simple Average	-0.19	4.61	168.56	9.46	0.83	9.55	0.91	9.56	0.94
Exponential Smoothing	-0.26	5.19	211.85	9.68	0.84	11.04	0.86	11.00	0.92
Regression	-0.13	4.64	169.37	9.32	0.81	9.32	0.91	9.32	0.91
AR(1)ULS	-0.18	4.53	161.84	8.74	0.83	9.63	0.89	9.59	0.94
AR(2)ULS	-0.16	4.55	163.77	8.42	0.82	10.15	0.91	10.11	0.97
MA(1)ULS	-0.18	4.53	162.02	8.69	0.83	9.32	0.89	9.32	0.94
MA(2)ULS	-0.19	4.52	159.86	8.42	0.83	9.32	0.89	9.32	0.94
ARMA(1,1)ULS	-0.18	4.61	168.35	9.32	0.83	9.33	0.89	9.32	0.94
ARMA(2,2) ULS	-0.15	4.56	164.24	8.39	0.83	9.34	0.89	9.32	0.94
ARMA(2,2) MLE	-0.17	4.58	166.15	8.45	0.85	9.40	0.89	9.36	0.93
Kalman Filter ARMA(2,2)	-0.03	4.56	164.56	9.01	0.81	9.46	0.90	9.43	0.93
MMAE ARMA(2,2)	-0.05	4.57	164.21	8.82	0.81	9.19	0.87	9.33	0.92

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	49	49.5

Table 83. Monte Carlo Simulation Results for Case 62

TRUE PARAMETERS

Phi1 =	0	Sample Size =	200
Phi2 =	0	Number of Predictions=	5
Theta1=	0.5	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.02	0.91	6.47	1.67	0.84	1.86	0.92	1.86	0.90
ARMA(2,2)InitialEstimates	-0.03	1.14	10.22	2.75	0.88	2.33	0.93	1.87	0.91
Naive	0.02	1.33	13.82	3.11	0.91	2.64	0.91	2.62	0.88
Moving Average (T=5)	-0.03	0.94	7.09	2.06	0.89	1.93	0.92	1.93	0.88
Moving Average (T=10)	-0.01	0.93	6.85	1.96	0.87	1.89	0.91	1.89	0.91
Simple Average	-0.01	0.92	6.57	1.89	0.87	1.87	0.93	1.87	0.90
Exponential Smoothing	0.00	1.10	9.32	2.25	0.86	1.99	0.89	1.98	0.82
Regression	-0.01	0.91	6.59	1.87	0.87	1.87	0.92	1.87	0.90
AR(1)ULS	-0.01	0.91	6.47	1.73	0.85	1.91	0.93	1.91	0.91
AR(2)ULS	-0.02	0.92	6.60	1.66	0.82	2.02	0.95	2.03	0.91
MA(1)ULS	-0.01	0.91	6.46	1.72	0.87	1.86	0.93	1.86	0.90
MA(2)ULS	-0.02	0.91	6.54	1.65	0.83	1.86	0.93	1.86	0.90
ARMA(1,1)ULS	-0.01	0.92	6.60	1.87	0.88	1.86	0.91	1.86	0.90
ARMA(2,2) ULS	-0.01	0.92	6.56	1.66	0.83	1.86	0.92	1.86	0.90
ARMA(2,2) MLE	-0.01	0.92	6.57	1.67	0.83	1.87	0.91	1.86	0.90
Kalman Filter ARMA(2,2)	-0.02	0.94	7.17	1.67	0.83	1.93	0.93	1.92	0.90
MMAE ARMA(2,2)	-0.03	0.95	7.40	1.87	0.88	2.08	0.95	2.09	0.92

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	49	51.5

Table 84. Monte Carlo Simulation Results for Case 63

TRUE PARAMETERS

Phi1 =	0	Sample Size =	200
Phi2 =	0	Number of Predictions=	5
Theta1=	-0.5	Noise Std. Deviation =	1
Theta2=	0	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.88	6.12	1.65	0.83	1.84	0.89	1.83	0.92
ARMA(2,2)InitialEstimates	-0.01	1.03	8.72	2.20	0.88	1.88	0.87	1.83	0.92
Naive	-0.02	1.14	10.18	2.04	0.87	2.62	0.91	2.60	0.94
Moving Average (T=5)	-0.04	1.01	7.88	2.00	0.84	2.11	0.91	2.11	0.91
Moving Average (T=10)	-0.02	0.97	7.20	1.93	0.84	1.99	0.90	1.99	0.92
Simple Average	-0.01	0.90	6.47	1.86	0.83	1.88	0.89	1.88	0.95
Exponential Smoothing	-0.03	1.02	8.24	1.90	0.85	2.18	0.85	2.17	0.90
Regression	-0.01	0.91	6.50	1.83	0.82	1.83	0.90	1.83	0.92
AR(1)ULS	-0.01	0.89	6.24	1.72	0.84	1.89	0.89	1.89	0.95
AR(2)ULS	-0.01	0.90	6.35	1.65	0.81	2.01	0.90	2.00	0.95
MA(1)ULS	-0.01	0.89	6.25	1.71	0.83	1.83	0.89	1.83	0.92
MA(2)ULS	-0.02	0.89	6.17	1.65	0.83	1.83	0.89	1.83	0.92
ARMA(1,1)ULS	0.00	0.89	6.32	1.78	0.84	1.84	0.89	1.84	0.92
ARMA(2,2) ULS	-0.01	0.89	6.21	1.64	0.82	1.84	0.89	1.83	0.92
ARMA(2,2) MLE	-0.01	0.90	6.41	1.66	0.83	1.84	0.88	1.84	0.92
Kalman Filter ARMA(2,2)	-0.01	0.89	6.29	1.66	0.82	1.85	0.88	1.85	0.94
MMAE ARMA(2,2)	-0.01	0.89	6.23	1.86	0.85	2.06	0.93	2.09	0.98

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	55.3	55.9

Table 85. Monte Carlo Simulation Results for Case 64

TRUE PARAMETERS

Phi1 =	0	Sample Size =	400
Phi2 =	0	Number of Predictions=	5
Theta1=	0.5	Noise Std. Deviation =	5
Theta2=	0.4	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.05	4.70	174.64	8.89	0.91	9.94	0.89	9.93	0.91
ARMA(2,2)InitialEstimates	0.07	5.29	222.93	11.36	0.90	11.12	0.88	9.96	0.90
Naive	0.15	6.95	387.90	15.44	0.89	14.07	0.92	14.01	0.95
Moving Average (T=5)	0.01	5.08	207.64	11.14	0.86	10.24	0.89	10.27	0.92
Moving Average (T=10)	0.04	4.97	193.43	10.49	0.87	10.02	0.87	10.03	0.92
Simple Average	0.02	4.84	184.81	10.03	0.87	9.95	0.89	9.95	0.91
Exponential Smoothing	0.08	5.79	267.80	11.95	0.85	10.62	0.89	10.62	0.93
Regression	0.02	4.85	185.31	9.95	0.87	9.95	0.89	9.95	0.91
AR(1)ULS	0.02	4.82	182.80	9.71	0.89	10.19	0.89	10.21	0.91
AR(2)ULS	0.02	4.74	177.01	9.12	0.91	10.69	0.94	10.63	0.91
MA(1)ULS	0.02	4.79	180.66	9.54	0.90	9.93	0.89	9.93	0.91
MA(2)ULS	-3.90	22.58	40376.12	64.48	0.45	9.93	0.89	9.93	0.91
ARMA(1,1)ULS	0.02	4.84	184.80	9.93	0.87	9.93	0.89	9.93	0.91
ARMA(2,2) ULS	0.08	5.35	357.26	10.92	0.87	10.14	0.87	10.03	0.90
ARMA(2,2) MLE	0.03	4.70	173.56	8.78	0.91	9.96	0.89	9.93	0.91
Kalman Filter ARMA(2,2)	0.05	4.93	192.96	9.80	0.90	10.11	0.88	10.28	0.91
MMAE ARMA(2,2)	0.05	4.91	191.14	9.50	0.89	9.57	0.86	9.60	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	37.7	38.5

Table 86. Monte Carlo Simulation Results for Case 65

TRUE PARAMETERS

Phi1 =	0	Sample Size =	400
Phi2 =	0	Number of Predictions=	5
Theta1=	-0.5	Noise Std. Deviation =	5
Theta2=	0.3	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	4.60	165.73	8.60	0.91	9.64	0.88	9.63	0.93
ARMA(2,2)InitialEstimates	0.00	5.12	204.60	9.76	0.91	9.65	0.84	9.63	0.93
Naive	-0.07	6.14	316.43	11.82	0.90	13.66	0.90	13.63	0.91
Moving Average (T=5)	0.00	5.04	207.38	10.64	0.88	10.66	0.91	10.66	0.93
Moving Average (T=10)	-0.13	4.87	190.92	10.13	0.91	10.15	0.91	10.15	0.92
Simple Average	-0.02	4.67	170.39	9.72	0.93	9.72	0.88	9.72	0.93
Exponential Smoothing	-0.07	5.40	246.28	10.56	0.88	10.97	0.88	10.96	0.91
Regression	0.02	4.69	170.64	9.64	0.93	9.64	0.90	9.64	0.93
AR(1)ULS	-0.02	4.66	171.34	9.33	0.91	9.91	0.88	9.90	0.93
AR(2)ULS	-0.01	4.64	169.76	8.85	0.91	10.37	0.91	10.31	0.93
MA(1)ULS	-0.02	4.64	169.47	9.18	0.91	9.63	0.88	9.63	0.93
MA(2)ULS	0.60	17.51	35803.02	58.68	0.78	9.63	0.88	9.63	0.93
ARMA(1,1)ULS	-0.01	4.67	170.37	9.64	0.93	9.63	0.88	9.63	0.93
ARMA(2,2) ULS	-0.03	4.62	168.97	8.91	0.91	9.70	0.88	9.65	0.92
ARMA(2,2) MLE	-0.02	4.57	164.53	8.52	0.92	9.65	0.88	9.63	0.93
Kalman Filter ARMA(2,2)	-0.03	4.68	171.72	9.50	0.89	9.69	0.88	9.68	0.93
MMAE ARMA(2,2)	-0.03	4.68	171.87	9.39	0.89	9.46	0.87	9.50	0.93

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	42.5	44.1

Table 87. Monte Carlo Simulation Results for Case 66

TRUE PARAMETERS

Phi1 =	0	Sample Size =	300
Phi2 =	0	Number of Predictions=	5
Theta1=	-1.3	Noise Std. Deviation =	5
Theta2=	-0.7	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.50	6.61	344.98	9.06	0.89	14.63	0.89	14.60	0.91
ARMA(2,2)InitialEstimates	-0.47	7.12	399.87	11.73	0.90	14.99	0.87	14.60	0.92
Naive	-0.39	8.86	644.82	11.74	0.90	20.88	0.92	20.78	0.91
Moving Average (T=5)	-0.85	8.53	548.84	15.53	0.83	17.73	0.91	17.65	0.94
Moving Average (T=10)	-0.80	8.13	504.60	15.20	0.83	16.39	0.88	16.36	0.91
Simple Average	-0.55	7.43	423.10	14.74	0.87	14.98	0.90	14.99	0.92
Exponential Smoothing	-0.49	8.61	595.84	12.96	0.92	19.15	0.90	19.05	0.90
Regression	-0.41	7.47	433.77	14.55	0.86	14.56	0.91	14.56	0.91
AR(1)ULS	-0.54	7.17	392.08	12.53	0.86	15.08	0.90	15.05	0.92
AR(2)ULS	-0.53	6.97	395.13	9.61	0.89	18.54	0.94	18.53	0.95
MA(1)ULS	-0.54	7.20	396.13	12.60	0.86	14.60	0.89	14.60	0.91
MA(2)ULS	-0.46	14.10	7005.33	38.73	0.66	14.60	0.89	14.60	0.91
ARMA(1,1)ULS	-0.94	7.63	463.24	14.17	0.88	15.43	0.88	15.39	0.89
ARMA(2,2) ULS	-0.21	7.30	504.84	10.74	0.89	15.94	0.88	15.67	0.91
ARMA(2,2) MLE	-0.47	6.58	339.85	9.04	0.87	14.67	0.89	14.60	0.90
Kalman Filter ARMA(2,2)	-0.77	7.04	387.43	11.65	0.89	15.44	0.89	14.97	0.93
MMAE ARMA(2,2)	-0.70	7.00	383.45	10.57	0.83	12.57	0.80	13.27	0.84

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	41.3	42.8

Table 88. Monte Carlo Simulation Results for Case 67

TRUE PARAMETERS

Phi1 =	0	Sample Size =	400
Phi2 =	0	Number of Predictions=	5
Theta1=	0.5	Noise Std. Deviation =	5
Theta2=	-0.3	Error Std. Deviation =	1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.01	4.58	164.07	8.46	0.92	9.70	0.89	9.70	0.88
ARMA(2,2)InitialEstimates	-0.01	5.91	284.62	13.16	0.86	10.73	0.89	9.77	0.90
Naive	0.21	7.04	394.35	16.67	0.88	13.75	0.86	13.69	0.92
Moving Average (T=5)	0.07	5.02	199.17	10.63	0.88	10.17	0.90	10.18	0.91
Moving Average (T=10)	-0.02	4.96	191.82	10.18	0.86	9.94	0.90	9.94	0.90
Simple Average	0.03	4.82	182.70	9.79	0.88	9.73	0.89	9.74	0.90
Exponential Smoothing	0.12	5.78	263.37	11.62	0.80	10.43	0.80	10.40	0.89
Regression	0.04	4.82	183.21	9.71	0.88	9.71	0.89	9.71	0.86
AR(1)ULS	0.02	4.69	172.71	8.83	0.88	9.94	0.91	9.96	0.91
AR(2)ULS	0.00	4.76	180.16	8.92	0.95	11.55	0.94	11.61	0.94
MA(1)ULS	0.02	4.72	175.33	8.91	0.89	9.69	0.89	9.69	0.88
MA(2)ULS	1.05	20.18	20789.53	74.44	0.61	9.69	0.89	9.69	0.88
ARMA(1,1)ULS	0.03	4.82	182.62	9.69	0.88	9.69	0.89	9.70	0.88
ARMA(2,2) ULS	0.03	4.85	190.89	11.32	0.92	9.98	0.90	9.93	0.89
ARMA(2,2) MLE	0.04	4.58	165.02	8.47	0.93	9.87	0.91	9.83	0.92
Kalman Filter ARMA(2,2)	-0.14	5.36	281.95	8.67	0.93	10.84	0.92	14.40	0.90
MMAE ARMA(2,2)	-0.12	5.55	297.28	8.42	0.93	9.36	0.88	9.58	0.72

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	35.6	37.6

Table 89. Monte Carlo Simulation Results for Case 68

TRUE PARAMETERS

Phi1 =	0	Sample Size =	400
Phi2 =	0	Number of Predictions=	5
Theta1=	0.5	Noise Std. Deviation =	1
Theta2=	0.4	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.02	0.92	6.68	1.69	0.90	1.97	0.91	1.96	0.90
ARMA(2,2)InitialEstimates	0.02	1.01	7.99	2.12	0.92	2.18	0.89	1.97	0.91
Naive	0.06	1.38	15.29	3.06	0.88	2.78	0.90	2.77	0.94
Moving Average (T=5)	0.01	1.00	8.03	2.20	0.88	2.02	0.87	2.03	0.92
Moving Average (T=10)	0.02	0.98	7.49	2.07	0.88	1.98	0.89	1.98	0.92
Simple Average	0.01	0.95	7.15	1.98	0.90	1.97	0.90	1.97	0.90
Exponential Smoothing	0.03	1.15	10.52	2.37	0.85	2.10	0.88	2.10	0.93
Regression	0.01	0.96	7.16	1.97	0.90	1.97	0.91	1.97	0.90
AR(1)ULS	0.01	0.95	7.06	1.92	0.90	2.02	0.91	2.02	0.90
AR(2)ULS	0.01	0.93	6.85	1.80	0.91	2.12	0.94	2.11	0.90
MA(1)ULS	0.01	0.94	6.98	1.88	0.90	1.96	0.90	1.96	0.90
MA(2)ULS	0.01	0.92	6.68	1.67	0.89	1.96	0.90	1.96	0.90
ARMA(1,1)ULS	0.01	0.96	7.16	1.97	0.90	1.96	0.90	1.96	0.90
ARMA(2,2) ULS	0.02	0.92	6.65	1.69	0.89	1.97	0.90	1.96	0.90
ARMA(2,2) MLE	0.02	0.92	6.64	1.70	0.90	1.97	0.90	1.96	0.90
Kalman Filter ARMA(2,2)	0.02	0.92	6.66	1.68	0.90	1.97	0.90	1.97	0.90
MMAE ARMA(2,2)	0.01	0.93	6.73	1.86	0.93	2.06	0.91	2.06	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	48.8	50

Table 90. Monte Carlo Simulation Results for Case 69

TRUE PARAMETERS

Phi1 =	0	Sample Size =	400
Phi2 =	0	Number of Predictions=	5
Theta1=	-0.5	Noise Std. Deviation =	1
Theta2=	0.3	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.01	0.90	6.36	1.67	0.90	1.91	0.88	1.91	0.93
ARMA(2,2)InitialEstimates	0.01	1.00	7.77	1.89	0.90	1.91	0.85	1.91	0.94
Naive	0.00	1.22	12.38	2.33	0.90	2.71	0.91	2.70	0.94
Moving Average (T=5)	0.01	1.00	8.09	2.11	0.88	2.11	0.90	2.11	0.94
Moving Average (T=10)	-0.01	0.97	7.43	2.00	0.90	2.01	0.90	2.01	0.94
Simple Average	0.01	0.92	6.61	1.92	0.93	1.92	0.89	1.92	0.93
Exponential Smoothing	0.00	1.07	9.58	2.09	0.88	2.17	0.90	2.17	0.92
Regression	0.02	0.93	6.62	1.91	0.91	1.91	0.89	1.91	0.93
AR(1)ULS	0.01	0.92	6.63	1.84	0.92	1.96	0.89	1.96	0.93
AR(2)ULS	0.01	0.92	6.58	1.74	0.90	2.06	0.91	2.05	0.94
MA(1)ULS	0.01	0.92	6.56	1.81	0.90	1.91	0.88	1.91	0.93
MA(2)ULS	0.01	0.90	6.38	1.66	0.90	1.91	0.88	1.91	0.93
ARMA(1,1)ULS	0.01	0.92	6.62	1.90	0.92	1.91	0.89	1.91	0.93
ARMA(2,2) ULS	0.01	0.90	6.36	1.66	0.91	1.91	0.88	1.91	0.93
ARMA(2,2) MLE	0.01	0.90	6.40	1.67	0.91	1.91	0.89	1.91	0.93
Kalman Filter ARMA(2,2)	0.01	0.91	6.43	1.67	0.90	1.92	0.88	1.91	0.93
MMAE ARMA(2,2)	0.01	0.91	6.43	1.87	0.92	2.05	0.90	2.06	0.94

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	51.2	54
MMAE ARMA(2,2)	85.6	85.6

Table 91. Monte Carlo Simulation Results for Case 70

TRUE PARAMETERS

Phi1 =	0	Sample Size =	400
Phi2 =	0	Number of Predictions=	5
Theta1=	-1.3	Noise Std. Deviation =	1
Theta2=	-0.7	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.30	13.48	1.69	0.88	2.91	0.87	2.91	0.94
ARMA(2,2)InitialEstimates	0.00	1.46	16.80	2.25	0.84	3.00	0.85	2.91	0.91
Naive	-0.04	1.71	24.54	2.30	0.92	4.14	0.91	4.13	0.92
Moving Average (T=5)	0.01	1.60	20.72	3.08	0.93	3.53	0.88	3.52	0.92
Moving Average (T=10)	-0.05	1.52	18.77	3.02	0.90	3.26	0.89	3.26	0.91
Simple Average	0.00	1.42	15.54	2.93	0.92	2.97	0.87	2.97	0.95
Exponential Smoothing	-0.03	1.66	22.96	2.56	0.93	3.80	0.90	3.79	0.92
Regression	0.02	1.43	15.69	2.90	0.90	2.90	0.87	2.90	0.94
AR(1)ULS	0.00	1.38	14.61	2.49	0.90	3.00	0.88	2.99	0.95
AR(2)ULS	0.01	1.37	14.85	1.83	0.88	3.65	0.94	3.64	0.98
MA(1)ULS	0.00	1.39	14.78	2.50	0.91	2.91	0.87	2.91	0.94
MA(2)ULS	0.00	1.33	13.89	1.76	0.89	2.91	0.87	2.91	0.94
ARMA(1,1)ULS	0.01	1.41	15.24	2.84	0.92	2.92	0.88	2.92	0.94
ARMA(2,2) ULS	0.00	1.30	13.59	1.68	0.88	2.91	0.84	2.91	0.94
ARMA(2,2) MLE	0.02	1.31	13.67	1.70	0.90	2.92	0.87	2.91	0.92
Kalman Filter ARMA(2,2)	0.02	1.31	13.49	1.71	0.91	2.92	0.86	2.92	0.94
MMAE ARMA(2,2)	0.02	1.30	13.50	2.04	0.98	3.02	0.88	3.06	0.97

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	54.8	51.2

Table 92. Monte Carlo Simulation Results for Case 71

TRUE PARAMETERS

Phi1 =	0.9	Sample Size =	200
Phi2 =	-0.6	Number of Predictions=	5
Phi3=	0.5	Noise Std. Deviation =	5
Theta1=	-0.8	Error Std. Deviation =	1
Theta2=	-0.5		
Theta3=	0.2		

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.23	12.54	1296.19	11.31	0.88	23.07	0.78	23.30	0.82
ARMA(2,2)InitialEstimates	0.01	10.74	938.94	12.09	0.91	21.38	0.86	21.96	0.82
Naive	0.12	10.50	845.13	13.42	0.86	23.91	0.88	24.80	0.95
Moving Average (T=5)	0.20	10.89	936.95	18.04	0.89	23.39	0.89	25.66	0.91
Moving Average (T=10)	0.84	10.85	963.07	20.24	0.90	23.91	0.91	25.60	0.94
Simple Average	-0.17	12.25	1196.38	23.29	0.89	24.14	0.90	24.60	0.88
Exponential Smoothing	0.14	10.36	819.80	14.97	0.91	22.24	0.86	24.18	0.94
Regression	-0.63	12.47	1223.70	22.42	0.89	22.42	0.85	22.42	0.84
AR(1)ULS	-0.15	11.74	1099.03	18.75	0.89	21.08	0.84	21.31	0.82
AR(2)ULS	0.13	11.21	1013.21	12.08	0.86	22.50	0.87	22.15	0.84
MA(1)ULS	-0.16	11.88	1128.40	19.28	0.90	22.99	0.87	22.98	0.85
MA(2)ULS	-0.62	13.39	1706.39	23.08	0.74	22.99	0.87	22.98	0.85
ARMA(1,1)ULS	-0.12	10.52	875.59	11.98	0.84	24.88	0.88	28.18	0.97
ARMA(2,2) ULS	-1.84	18.36	7330.62	23.11	0.84	27.49	0.82	24.89	0.86
ARMA(2,2) MLE	0.09	10.24	858.41	10.37	0.83	21.48	0.85	22.47	0.86
Kalman Filter ARMA(2,2)	0.29	10.20	823.27	15.26	0.90	20.74	0.85	21.62	0.91
MMAE ARMA(2,2)	0.13	10.36	843.02	13.19	0.85	15.56	0.75	16.64	0.80

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	47.6	49.2

Table 93. Monte Carlo Simulation Results for Case 72

TRUE PARAMETERS

Phi1 =	0.9	Sample Size =	200
Phi2 =	-0.6	Number of Predictions=	5
Phi3=	0.5	Noise Std. Deviation =	1
Theta1=	-0.8	Error Std. Deviation =	0.1
Theta2=	-0.5		
Theta3=	0.2		

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.05	2.47	51.01	2.68	0.95	4.67	0.78	4.66	0.82
ARMA(2,2)InitialEstimates	0.00	2.13	36.99	2.99	0.99	4.44	0.86	4.42	0.84
Naive	0.02	2.10	33.66	2.65	0.88	4.76	0.89	4.94	0.95
Moving Average (T=5)	0.04	2.17	37.20	3.59	0.89	4.67	0.88	5.12	0.92
Moving Average (T=10)	0.17	2.17	38.22	4.04	0.89	4.77	0.90	5.11	0.94
Simple Average	-0.04	2.45	47.60	4.65	0.89	4.82	0.89	4.91	0.89
Exponential Smoothing	0.03	2.07	32.64	2.98	0.92	4.43	0.87	4.82	0.94
Regression	-0.13	2.49	48.74	4.48	0.89	4.48	0.86	4.48	0.84
AR(1)ULS	-0.03	2.34	43.71	3.74	0.89	4.21	0.84	4.25	0.83
AR(2)ULS	0.03	2.06	34.37	2.34	0.85	4.47	0.90	4.46	0.88
MA(1)ULS	-0.04	2.37	44.88	3.84	0.89	4.59	0.86	4.59	0.86
MA(2)ULS	-0.03	2.23	40.90	2.29	0.84	4.59	0.86	4.59	0.86
ARMA(1,1)ULS	0.14	2.07	32.68	2.99	0.90	4.34	0.89	4.86	0.96
ARMA(2,2) ULS	-0.04	2.12	37.35	2.15	0.84	4.57	0.87	4.58	0.86
ARMA(2,2) MLE	-0.02	2.06	36.28	2.27	0.92	4.41	0.86	4.55	0.86
Kalman Filter ARMA(2,2)	0.06	2.06	35.11	2.03	0.85	4.35	0.87	4.46	0.87
MMAE ARMA(2,2)	0.05	2.04	33.94	2.25	0.89	4.22	0.86	4.50	0.90

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	53.6	52.8

Table 94. Monte Carlo Simulation Results for Case 73

TRUE PARAMETERS

Phi1 =	0.4	Sample Size =	200
Phi2 =	0.3	Number of Predictions=	5
Phi3=	0.1	Noise Std. Deviation =	5
Theta1=	0.7	Error Std. Deviation =	1
Theta2=	-0.3		
Theta3=	0.4		

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.03	4.62	169.61	9.16	0.87	10.19	0.93	9.96	0.93
ARMA(2,2)InitialEstimates	-0.02	5.88	287.09	15.32	0.90	9.88	0.91	9.76	0.92
Naive	0.03	6.55	340.58	16.35	0.90	14.85	0.90	13.74	0.89
Moving Average (T=5)	-0.12	4.98	198.95	10.69	0.89	10.44	0.92	10.49	0.91
Moving Average (T=10)	0.03	4.85	189.53	10.27	0.89	10.12	0.94	10.14	0.92
Simple Average	-0.01	4.75	179.08	9.93	0.88	9.85	0.94	9.86	0.93
Exponential Smoothing	-0.01	5.44	236.03	11.45	0.86	11.20	0.88	10.66	0.86
Regression	-0.01	4.75	179.42	9.78	0.88	9.78	0.93	9.78	0.92
AR(1)ULS	-0.01	4.68	174.22	9.07	0.86	9.66	0.93	10.03	0.93
AR(2)ULS	-0.02	4.62	167.68	8.52	0.83	10.31	0.92	10.87	0.93
MA(1)ULS	-0.01	4.72	177.35	9.24	0.86	9.75	0.94	9.75	0.92
MA(2)ULS	0.00	4.71	175.38	8.66	0.82	9.75	0.94	9.75	0.92
ARMA(1,1)ULS	0.01	4.75	179.00	9.68	0.87	9.76	0.93	9.76	0.92
ARMA(2,2) ULS	-0.32	5.06	584.94	11.17	0.82	10.49	0.93	10.07	0.92
ARMA(2,2) MLE	-0.04	4.81	191.72	8.86	0.83	10.73	0.94	9.89	0.93
Kalman Filter ARMA(2,2)	-0.05	4.77	182.01	9.09	0.84	10.04	0.94	11.08	0.94
MMAE ARMA(2,2)	-0.05	4.80	187.78	8.86	0.83	9.32	0.90	9.46	0.89

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	49.2	44.8

Table 95. Monte Carlo Simulation Results for Case 74

TRUE PARAMETERS

Phi1 =	0.4	Sample Size =	200
Phi2 =	0.3	Number of Predictions=	5
Phi3=	0.1	Noise Std. Deviation =	1
Theta1=	0.7	Error Std. Deviation =	0.1
Theta2=	-0.3		
Theta3=	0.4		

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	-0.01	0.91	6.64	1.85	0.88	2.02	0.92	1.97	0.93
ARMA(2,2)InitialEstimates	0.00	1.17	11.28	4.07	0.97	1.96	0.90	1.93	0.92
Naive	0.00	1.29	13.34	3.24	0.90	2.94	0.90	2.72	0.88
Moving Average (T=5)	-0.03	0.99	7.82	2.11	0.89	2.06	0.93	2.07	0.88
Moving Average (T=10)	0.00	0.96	7.44	2.03	0.88	2.00	0.92	2.01	0.90
Simple Average	-0.01	0.94	7.03	1.97	0.86	1.95	0.93	1.95	0.92
Exponential Smoothing	0.00	1.08	9.28	2.27	0.86	2.22	0.89	2.11	0.85
Regression	-0.01	0.94	7.05	1.94	0.87	1.94	0.92	1.94	0.92
AR(1)ULS	-0.01	0.93	6.82	1.79	0.85	1.91	0.91	1.99	0.92
AR(2)ULS	-0.01	0.91	6.55	1.68	0.84	2.05	0.90	2.16	0.93
MA(1)ULS	-0.01	0.93	6.95	1.83	0.86	1.93	0.92	1.93	0.92
MA(2)ULS	-0.01	0.93	6.88	1.68	0.81	1.93	0.92	1.93	0.92
ARMA(1,1)ULS	0.00	0.99	7.88	2.31	0.89	2.13	0.92	2.12	0.89
ARMA(2,2) ULS	0.00	0.91	6.54	1.69	0.81	1.93	0.92	1.93	0.92
ARMA(2,2) MLE	0.00	0.92	6.55	1.84	0.83	2.09	0.95	1.96	0.92
Kalman Filter ARMA(2,2)	0.00	0.92	6.65	1.68	0.83	1.95	0.94	1.98	0.92
MMAE ARMA(2,2)	0.00	0.91	6.63	1.88	0.89	2.13	0.96	2.18	0.93

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	52.4	52.8

Table 96. Monte Carlo Simulation Results for Case 75

TRUE PARAMETERS

Phi1 =	0.7	Sample Size =	200
Phi2 =	-0.8	Number of Predictions=	5
Phi3=	0.3	Noise Std. Deviation =	1
Theta1=	0.6	Error Std. Deviation =	0.1
Theta2=	0.3		
Theta3=	-0.8		

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.96	30.56	4.23	0.88	3.25	0.84	3.29	0.77
ARMA(2,2)InitialEstimates	-0.01	1.44	16.92	3.10	0.92	3.04	0.88	3.45	0.92
Naive	-0.24	2.26	42.42	4.62	0.89	5.04	0.87	4.11	0.89
Moving Average (T=5)	-0.02	1.73	23.43	3.48	0.88	3.19	0.84	3.62	0.90
Moving Average (T=10)	-0.03	1.74	23.54	3.51	0.88	3.26	0.84	3.53	0.90
Simple Average	-0.01	1.73	23.12	3.44	0.88	3.34	0.86	3.43	0.89
Exponential Smoothing	-0.15	1.92	30.66	4.04	0.88	3.33	0.82	3.60	0.90
Regression	0.00	1.73	23.16	3.39	0.87	3.39	0.86	3.39	0.89
AR(1)ULS	-0.02	1.74	23.52	3.42	0.88	3.56	0.88	3.26	0.87
AR(2)ULS	0.00	1.27	13.07	2.00	0.85	5.43	0.99	3.49	0.92
MA(1)ULS	-0.02	1.72	22.75	3.28	0.88	3.37	0.86	3.37	0.88
MA(2)ULS	-0.01	1.64	20.81	2.38	0.86	3.37	0.86	3.37	0.88
ARMA(1,1)ULS	-0.01	1.74	23.40	3.45	0.88	3.36	0.86	3.37	0.88
ARMA(2,2) ULS	0.01	1.38	14.92	2.11	0.87	3.02	0.88	3.27	0.89
ARMA(2,2) MLE	0.01	1.30	13.73	2.04	0.88	2.79	0.83	3.18	0.89
Kalman Filter ARMA(2,2)	0.01	1.74	24.88	2.04	0.86	4.67	0.85	3.21	0.88
MMAE ARMA(2,2)	0.01	1.73	24.58	2.13	0.86	2.73	0.64	3.00	0.85

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	24.4	24.4

Table 97. Monte Carlo Simulation Results for Case 76

TRUE PARAMETERS

Phi1 =	0.7	Sample Size =	200
Phi2 =	-0.8	Number of Predictions=	5
Phi3=	0.3	Noise Std. Deviation =	5
Theta1=	0.6	Error Std. Deviation =	1
Theta2=	0.3		
Theta3=	-0.8		

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.01	9.89	773.32	18.98	0.86	16.31	0.84	16.50	0.76
ARMA(2,2)InitialEstimates	0.00	7.57	469.86	14.96	0.88	15.19	0.86	17.17	0.92
Naive	-1.21	11.35	1065.85	23.21	0.89	25.30	0.87	20.67	0.90
Moving Average (T=5)	-0.08	8.69	590.13	17.48	0.89	16.03	0.84	18.19	0.90
Moving Average (T=10)	-0.12	8.75	593.35	17.63	0.86	16.36	0.85	17.71	0.90
Simple Average	-0.01	8.71	582.07	17.26	0.88	16.74	0.87	17.21	0.90
Exponential Smoothing	-0.72	9.63	771.21	20.25	0.88	16.73	0.81	18.07	0.90
Regression	0.02	8.69	583.15	17.00	0.88	17.00	0.86	17.00	0.89
AR(1)ULS	-0.10	8.76	591.81	17.16	0.88	17.85	0.88	16.37	0.87
AR(2)ULS	0.03	6.43	334.31	10.18	0.87	27.12	0.99	17.53	0.92
MA(1)ULS	-0.07	8.62	572.50	16.45	0.88	16.92	0.88	16.92	0.88
MA(2)ULS	0.10	10.46	1566.14	26.29	0.87	16.92	0.88	16.92	0.88
ARMA(1,1)ULS	-0.01	8.71	582.27	17.02	0.87	16.92	0.87	16.91	0.88
ARMA(2,2) ULS	-0.28	7.47	481.35	13.78	0.86	15.55	0.85	16.48	0.89
ARMA(2,2) MLE	0.05	6.50	342.68	9.94	0.84	13.79	0.85	15.92	0.89
Kalman Filter ARMA(2,2)	0.07	8.77	588.65	14.65	0.88	19.81	0.88	16.70	0.88
MMAE ARMA(2,2)	0.06	8.61	570.10	11.76	0.71	11.86	0.64	11.93	0.77

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	25.6	27.2

Table 98. An Example of Monte Carlo Simulation Results Showing Robustness of MMAE

TRUE PARAMETERS

Phi1 =	0	Sample Size =	400
Phi2 =	0	Number of Predictions=	5
Theta1=	1.3	Noise Std. Deviation =	1
Theta2=	-0.7	Error Std. Deviation =	0.1

STATISTICS

	ME	MAE	SSE	PIW1MSE	PIC1MSE	PIW3MSE	PIC3MSE	PIW5MSE	PIC5MSE
TRUE	0.00	1.31	13.60	1.73	0.92	2.95	0.90	2.95	0.87
ARMA(2,2)InitialEstimates	0.83	45.46	27401.17	54.61	0.76	167.28	1.00	183.32	1.00
Naive	0.07	2.21	39.00	5.43	0.89	4.18	0.86	4.15	0.85
Moving Average (T=5)	0.02	1.53	17.87	3.25	0.88	2.99	0.89	3.00	0.91
Moving Average (T=10)	0.01	1.51	17.36	3.10	0.88	2.97	0.89	2.97	0.88
Simple Average	0.01	1.48	16.98	2.98	0.88	2.95	0.90	2.95	0.86
Exponential Smoothing	0.04	1.79	25.02	3.67	0.84	3.08	0.82	3.07	0.84
Regression	0.01	1.49	17.00	2.96	0.88	2.96	0.89	2.96	0.86
AR(1)ULS	0.01	1.42	15.76	2.52	0.86	3.03	0.92	3.03	0.89
AR(2)ULS	0.01	1.40	15.29	1.83	0.89	3.67	0.97	3.68	0.97
MA(1)ULS	0.01	1.44	15.99	2.53	0.86	2.95	0.90	2.95	0.86
MA(2)ULS	0.00	1.36	14.56	1.76	0.89	2.95	0.90	2.95	0.86
ARMA(1,1)ULS	0.01	1.48	16.97	2.94	0.88	2.95	0.90	2.95	0.86
ARMA(2,2) ULS	0.85	44.32	27181.30	52.35	0.74	160.89	0.99	176.24	0.98
ARMA(2,2) MLE	0.86	44.33	27181.42	52.37	0.74	160.89	0.99	176.24	0.98
Kalman Filter ARMA(2,2)	-0.02	1.51	22.15	1.75	0.90	3.21	0.89	3.77	0.86
MMAE ARMA(2,2)	-0.03	1.56	25.45	2.14	0.92	3.38	0.90	3.49	0.86

PERCENT OF ESTIMATES BETTER THAN MLE OUT OF 1000 REPLICATIONS

	MAE	SSE
MMAE ARMA(2,2)	85.6	85.6

BIBLIOGRAPHY

1. Anderson, O. D. *Time Series Analysis and Forecasting Box-Jenkins Approach.* London and Boston: Butterwoths, 1977.

2. Ansley C. F. and Kohn R. " A Structured State Space Approach to Computing The Likelihood of an ARIMA Process and Its Derivatives," *Journal of Statistical Computation,* Vol. 21, NO. 2, 1985, 135.

3. Box George E. P. Jenkins Gwilyn M and Reinsel Gregory C. *Time Series Analysis.* New Jersey: Prentice Hall, Upper Saddle River, 1994.

4. Chang, C. B. and Athans, M. "State Estimation for Discrete Systems with Switching Parameters," *IEEE Transactions on Aerospace and Electronics Systems,* Vol. AES-4, No. 3, May 1978, 418-425.

5. Chang, C. B. and Athans, M. "Hypothesis Testing And State Estimation For Discrete Systems With Finite-Valued Switching Parameters" *MIT Electronics Systems Lab. Pub.* Vol. ESL-P-758, June 1977.

6. Chui C. K and Chen G. *Kalman Filtering With Real Time Applications.* New York: Springer Verlag, 1991.

7. Dent, Warren and Min An-Sin." A Monte Carlo Study of Autoregressive Integrated Moving Average Processes" *Journal of Econometrics,* Vol. 7, 1978, 232-55.

8. Gallagher Mark A. *Identification of The Initial Transient in Discrete-Event Simulation Output Using the Kalman Filter.* Dissertation, AFIT/DS/ENS/92-1, School of Engineering, Air Force Institute of Technology, Wright-Patterson AFB OH, December 1992.

9. Gallagher Mark A. Class Handouts, OPER 684, Quantitative Forecasting Techniques. School of Engineering, Air Force Institute of Technology, Wright-Patterson AFB OH, August 2000.

10. Gardner, G. and others." An Algorithm for Exact Maximum Likelihood Estimation of Autoregressive-Moving Average Models by Means of Kalman Filtering" *Applied Statistics,* 310-322.

11. Hanlon Peter D. *Failure Identification Using Multiple Model Adaptive Estimation For The Lambda flight Vehicle.* Thesis, AFIT/GE/ENG/92-D19, Air Force Institute of Technology, Wright-Patterson AFB OH, December 1992.

12. Harvey, Andrew C. *Forecasting, Structural Time Series Models and The Kalman Filter.* New York: Cambridge University Press, 1989.

13. Kyger David W. and Maybeck, Peter S. "Reducing Lag in Virtual Displays Using Multiple Model Adaptive Estimation," *IEEE Transactions On Aerospace and Electronics Systems,* Vol. 34, No. 4, October 1998, 1237.

14. Magill, D.T."Optimal Adaptive Estimation of Sample Stochastic Process," *IEEE Conference On Automatic Control* Vol. AC-10, No. 4, October 1965, 434-439.

15. Makridakis, Spyros and others. *Forecasting Methods And Applications* (3rd Edition). New York: John Wiley And Sons,Inc, 1998.

16. Maybeck, Peter S. *Stochastic Models, Estimation and Control, 1.* New York: Academic Press, Inc., 1979.

17. Maybeck, Peter S. *Stochastic Models, Estimation and Control, 2.* New York: Academic Press, Inc., 1979.

18. Maybeck, Peter S. *Stochastic Models, Estimation and Control, 3.* New York: Academic Press, Inc., 1979.

19. Miller Mikel M. *Modified Multiple Model Adaptive Estimation for Simultaneous Parameter and State Estimation.* Dissertation, AFIT/DS/ENG/98-2, School of Engineering, Air Force Institute of Technology, Wright-Patterson AFB OH, March 1998.

20. Morgan, Bruce W. *An Introduction to Bayesian Statistical Decision Processes.* New Jersey: Prentice-Hall Inc., 1968.

21. Netzer Alan S. *Characteristics of Bayesian Multiple Model Adaptive Estimation For Tracking Airborne Targets.* Thesis, AFIT/GAE/ENG/85-D2, Air Force Institute of Technology, Wright-Patterson AFB OH, December 1985.

22. Spall, James C. *Bayesian Analysis of Time Series and Dynamic Models.* New York: Marcel Dekker, Inc, 1998.

23. Sorenson, Harold W. *Kalman Filtering Theory and Application.* New York: IEEE Press, 1985.

24. Vasquez, Juan R. *New algorithms for Moving-Bank Multiple Model Adaptive Estimation.*Dissertation, AFIT/DS/ENG/98-10, Air Force Institute of Technology, Wright-Patterson AFB OH, May 1998.

VITA

1st Lieutenant Ibrahim DULGER was born in Samsun, Türkiye. He graduated from Maltepe Military High School in Izmir, in June 1990. In the same year he entered the Air Force Academy in İstanbul and graduated as a 2nd Lieutenant with a Bachelor of Science degree in Industrial Engineering on 30 August 1994. Then, he went to Çiğli Air Force Base, Izmir for undergraduate pilot training (UPT). After graduating from UPT in March 1996, he was assigned to 12th Airlift Base, Kayseri, Türkiye as a C-160 pilot. While he was on this assignment, he was chosen for Air Force Institute of Technology (AFIT). In August 1999 he entered the Graduate School of Engineering, Operrations Research Department, AFIT.After graduation he will return to his previous assignment, as a pilot. He is married.

REPORT DOCUMENTATION PAGE			*Form Approved* *OMB No. 074-0188*

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From – To)*
20-03-2001	Master's Thesis	Jun 2000-Mar 2001

4. TITLE AND SUBTITLE
MULTIPLE MODEL ADAPTIVE ESTIMATION FOR TIME SERIES ANALYSIS

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Ibrahim DULGER, 1st. Lt. TUAF

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S)
Air Force Institute of Technology
Graduate School of Engineering and Management (AFIT/EN)
2950 P Street, Building 640
WPAFB OH 45433-8865

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT/GOR/ENS/01M-07

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
N/A

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Multiple Model Adaptive Estimation (MMAE) is a Bayesian technique that applies a bank of Kalman filters to predict future observations. Each Kalman filter is based on a different set of parameters and hence produces different residuals. The likelihood of each Kalman filter's prediction is determined by a magnitude of the residuals. Since some researchers have obtained good forecasts using a single Kalman filter, we tested MMAE's ability to make time series predictions. Our Kalman filters have a dynamics model based on a Box-Jenkins Auto-Regressive Moving Average (ARMA) model and a measure model with additive noise. The time-series prediction is based on the probabilistic weighted Kalman filter predictions. We make a probability interval about that estimate also based on the filter probabilities. In a Monte Carlo analysis, we test this MMAE approach and report the results based on many different criteria. Our analysis tests the robustness of the approach by testing its ability to make predictions when the Kalman filter dynamics models did not match the data generation time-series model. Our analysis indicates benefits in applying multiple model adaptive estimation for time series analysis.

15. SUBJECT TERMS
Multiple Model Adaptive Estimation, Kalman Filtering, Time Series Analysis, Forecasting, Economic Forecasting, Maximum Likelihood Estimation, Monte Carlo Method, Least Square Method

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
					Lt Col Mark A Gallagher
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			**19b. TELEPHONE NUMBER** *(Include area code)*
U	U	U	UU	153	(937) 255-6565, ext 4335, Mark.Gallagher@afit.edu

	Form Approved *OMB No. 074-0188*